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DEWATERING OF SEWAGE SLUDGE ON GRANULAR MATERIALS

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John H. Nebiker

Thomas G. Sanders

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this document may be better
studied on microfiche

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ABSTRACT

A theoretical analysis of gravity dewatering of wastewater sludge is made, and an equation describing the drainage rate developed. The equation relates the depth of sludge with time, using the parameters of solids content, specific resistance, coefficient of compressibility, and dynamic viscosity and density of the filtrate. Extensive bench-scale tests substantiated the theory. The concept of media factor is introduced to account for the role of the supporting media. Potential applications of the equation developed are discussed.

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TABLE OF CONTENTS

Chapter

I Introduction 1

II Historical development and earlier research 8

III Recent research 16

IV Derivation of specific resistance 27

V Derivation of mathematical model 39

VI Experimental determination of specific resistance and coefficient of compressibility 45

VII Experimental determination of gravity dewatering rates . . 51

VIII Discussion of results 62

IX Significance of results and conclusions 81

Appendix I Derivation of the Mathematical Model for a Settled Sludge 85

Appendix II Experimental Data 94

Appendix III Graphical Representation of Experimental Data 104

Appendix IV Computer Program for Calculation of Specific Resistance 123

Appendix V Bibliography 125

LIST OF TABLES

Table Page

1. Characteristics of Sludges Used 59

2. Classification of Experiments 60

3. Experimental Data 65

4. Physical Characteristics of Supporting Sand 73

A1. Solids Content Data from Experiments I-IV 94

A2. Sand Data from Sieve Analysis 95

A3. Drainage Data from Experiment I 96

A4. Drainage Data from Experiment II 97

A5. Drainage Data from Experiment III 98

A6. Drainage Data from Experiment IV 99

A7. Drainage Data from Experiment V 101

A8. Drainage Data with Piezometer Readings from Experiment VI . 103

LIST OF FIGURES

Figure Page

1. Predicted sludge volumes for primary and secondary wastewater treatment in the United States (16) 2

2. Weight and volume reduction of 1 kg. of wastewater sludge at 5% initial solids content. The water reduction processes indicated are generally effective over the ranges of solids contents shown (17) 4

3. Loading rate of wastewater sludge at various treatment plants in the United States. Shown as a function of the initial solids content, the points and straight line represent the results of Haseltine. The curve is from Vater (1) 12

4. A plot of Haseltine's data of net bed loading versus initial solids content. Data is from American plants. Net bed loading is defined as gross bed loading multiplied by the water content of the sludge removed (9) (1) 14

5. A comparison of a typical soil permeameter to Jeffrey's drainometer (11) 17

6. Jeffrey's drainage data. The curves represent Jeffrey's empirically derived equation (11) 20

7. A plot of sludge moisture content w versus $(w/100)^3/(1-w/100)$ to prove that K/L is not constant with a change in moisture content 21

8. The Swanwick relationship of bed loading to specific resistance. Each point represents the mean of a group of loading values (1) (33) 23

9. A plot of data accumulated by Logsdon and Jeffrey with dewatering time by gravity versus dewatering time using a vacuum. Solids content of sludge range from 2.08% in Test 6 to 3.13% in Test 3 (15) 25

10. A schematic representation of filtration of a compressible material on a porous medium. As the filtrate drains from the sludge, solids are left behind at the sludge-filter interface forming a filter cake. The specific gravity of sludge particles is about 1.2, thus the sludge particles, mostly of micron size, will settle very slowly, justifying the assumption that the rate of build-up of the filter cake is directly proportional to the volume of filtrate only, and not to the settling velocity of the particles 34

11. A typical graph used to determine the specific resistance of a wastewater sludge at three different vacuums. From Experiment IV 36

12. Schematic representation of the different terms used in the derivation of the mathematical model 41

13. Apparatus to determine the specific resistance of compressible sludges 46

14. A plot of specific resistance versus head (converted from inches of Hg to cm of water) with a resulting slope equal to the coefficient of compressibility. From Experiment IV 49

15. Gravity dewatering apparatus with piezometer tubes. All dimensions are in cm 52

16. Gravity dewatering columns
The choice of nine columns is dictated by statistical considerations to allow simultaneous experiments with two variables. Here (Experiment V) the variables include three types of sand and three different filling depths (11, 41, and 81 cm). A sludge solids-supernatant interface is being observed 53

17. A sludge cake formed resulting from gravity dewatering only. From Experiment V 57

18. Experiment IV. Triplicate tests with 20 cm of sludge on Franklin sand. 69

19. Experiment IV. Triplicate tests with 20 cm of sludge on Ottawa sand 71

20. Experiment IV. Triplicate tests with 20 cm of sludge on Hermitage sand 72

21. Experiment V. 11 cm of sludge applied on three different sands 74

22. Experiment V. 41 cm of sludge applied on three different sands 75

23. Experiment V. 81 cm of sludge applied on three different sands 76

24. A plot of the 95% confidence interval with data from column 1, 2, and 3 in Experiment IV 78

25. Filtrate collection during the first 3 days showing relationship of filtrate to supporting media (sand). From left to right, the first and fourth graduated cylinders exhibit a small but clear volume of filtrate; the third and sixth, a larger volume of filtrate but one of much higher turbidity. The second and fifth cylinders show intermediate results. Respectively, the sands used are types F, O, and H, representing effective sand sizes of 0.16, 0.60, and 0.78 mm 79

A1. Specific resistance determinations for Experiment I . . 104

A2. Specific resistance determinations for Experiment II . 105

A3. Specific resistance determinations for Experiment III . 106

A4. Specific resistance determinations for Experiment IV. . 107

A5. Specific resistance determinations for Experiment V . . 108

A6. Specific resistance determinations for Experiment VI . 109

A7. Coefficient of compressibility determinations for Experiments I-VI 110
A8. Drainage plots for columns 1-9 of Experiment I. 111
A9. Drainage plots for columns 1-9 of Experiment II 112
A10. Drainage plots for columns 1-3 of Experiment III. 113
A11. Drainage plots for columns 4-6 of Experiment III. 114
A12. Drainage plots for columns 7-9 of Experiment III. 115
A13. Drainage plots for columns 1-3 of Experiment IV 116
A14. Drainage plots for columns 4-6 of Experiment IV 117
A15. Drainage plots for columns 7-9 of Experiment IV 118
A16. Drainage plots for columns 1-3 of Experiment V. 119
A17. Drainage plots for columns 4-6 of Experiment V. 120
A18. Drainage plots for columns 7-9 of Experiment V. 121
A19. Drainage plot for Experiment VI 122

LIST OF ABBREVIATIONS AND SYMBOLS

A = area (L^2)

A_p = cross sectional area of pores (L^2)

B = constant ($M^0L^0T^0$)

b = slope from t/V versus V plot ($T\ L^{-6}$)

C = specific resistance when $\sigma = 0$ (T^2M^{-1})

D = diameter (L)

d = number of days drainometer operates (T)

F = force ($M\ L\ T^{-2}$)

F = % solids content before filtration ($M^0L^0T^0$)

F_o = % solids content before filtration ($M^0L^0T^0$)

F_f = % solids content after filtration ($M^0L^0T^0$)

f = weight per unit volume of filtrate ($M\ L^{-2}\ T^{-2}$)

f_1 = friction factor ($M^0L^0T^0$)

g = acceleration due to gravity ($L\ T^{-2}$)

H_o = initial head (L)

H = head @ time t = t (L)

h = head @ time t = t (L)

h_c = head associated with specific resistance R_c (L)

h_f = head loss (L)

h_{hg} = height of mercury (L)

h_t = initial head minus thickness of sludge (L)

i = ordinate intercept (T)

K = permeability coefficient ($L\ T^{-1}$)

K = constant ($M^0L^0T^0$)

k = constant ($L\ T^{-1}$)

L = length

M = mass (M)

m = number of minutes vacuum operates (T)

N = number of particles ($M^0 L^0 T^0$)

ΔP = pressure ($M L^{-1} T^{-2}$), ($F L^{-2}$)

ΔP_c = pressure associated with specific resistance R_c ($M L^{-1} T^{-2}$)

P = wetted perimeter (L)

R = specific resistance ($T^2 M^{-1}$)

R_c = specific resistance with pressure loss P_c or head loss h_c ($T^2 M^{-1}$)

IR = Reynolds number ($M^0 L^0 T^0$)

R_f = resistance of non-compressible material ($T^2 M^{-1}$)

R_h = hydraulic radius (L)

S = solids content ($M^0 L^0 T^0$)

S_{hg} = specific gravity of mercury ($M^0 L^0 T^0$)

S_o = initial solids content ($M^0 L^0 T^0$)

S_p = surface area (L^2)

T = time (T)

t_t = time when $h_t = h$ (T)

V = volume (L^3)

V = final sludge depth (used in Figure 5 only) (L)

$\bar{V}$ = mean velocity ($L T^{-1}$)

V_o = initial sludge depth (L)

V_p = volume of one particle (L^3)

V_s = superficial velocity ($L T^{-1}$)

V_{tot} = total volume (L^3)

V_v = volume voids (L^3)

v = volume of cake deposited per unit volume of filtrate $(M^0L^0T^0)$

W_o = total quantity of liquor discharged from drainometer at time the quantity of discharge over a 24 hour period is equal to one per cent of total discharge (L^3)

W_{tot_f} = weight of filtered sludge $(M\ L\ T^{-2})$

W_{tot_o} = weight of unfiltered sludge $(M\ L\ T^{-2})$

W_{ts} = weight of total solids $(M\ L\ T^{-2})$

W_w = weight of water $(M\ L\ T^{-2})$

W_1 = quantity of filtrate @ time t (L^3)

w = % water content $(M^0L^0T^0)$

Y = gross bed loading of sludge solids $(M\ L^{-2}\ T^{-1})$

Y' = bed loading of sludge solids $(M\ L^{-2}\ T^{-1})$

Z = net bed loading of sludge solids $(M\ L^{-2}\ T^{-1})$

γ = specific weight of water $(M\ L^{-2}\ T^{-2})$

γ_t = specific weight of sludge $(M\ L^{-2}\ T^{-2})$

ϵ = porosity, V_v/V_{tot} $(M^0L^0T^0)$

μ = dynamic viscosity $(M\ L^{-1}\ T^{-1})$

ρ = mass density $(M\ L^{-3})$

σ = coefficient of compressibility $(M^0L^0T^0)$

CHAPTER I

INTRODUCTION

The rapid growth of population and industry, coupled with an increasing interest of state and Federal agencies concerned with water pollution abatement, has directed much attention to the problem of wastewater sludge treatment. With the advent of tertiary treatment to reduce pollutants in the river , a sludge will accumulate at the treatment plants similar to water treatment sludge and as difficult to dewater. This new source of sludge, along with the increasing volume of primary and secondary sludges, are major problems confronting the environmental engineer. The total residue produced from primary and secondary treatment alone (Figure 1) will amount to 150 million gallons a day in the United States by 1980 if the current trend is unchanged (16).

The methods used in the past to dewater and dispose of sludge will have to be re-evaluated and updated in order to efficiently deal with the staggering sludge volume of the future. If there is no further development in facilities or techniques, the cost of dewatering and disposal of the sludge may surpass the damage cost incurred if there were no treatment at all. In other words, the cost of pollution abatement may far surpass the monetary value which the public places on the resulting recreational and esthetic benefits, as well as the benefits derived from improved water quality.

Ultimate disposal of wastewater sludge may be achieved by hauling or pumping the sludge out to sea. If access to the ocean is unavailable, disposal of the sludge may be achieved by spreading on farmland

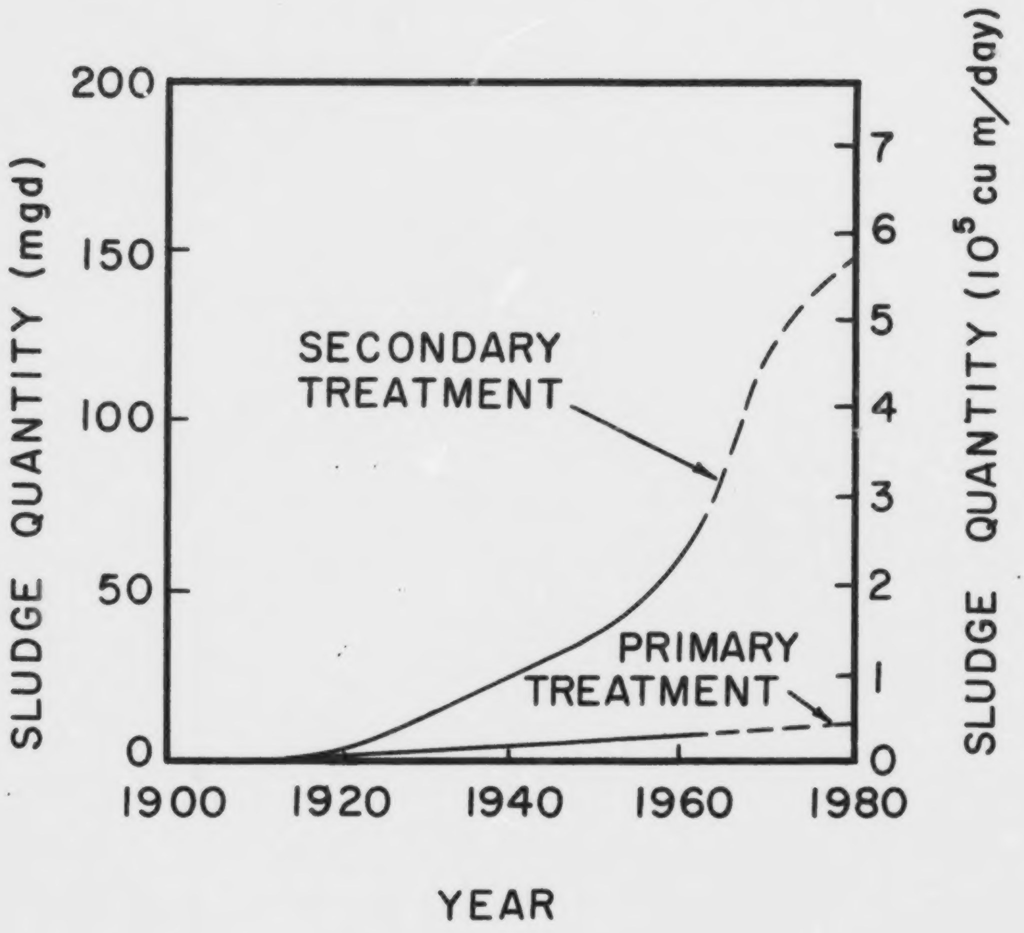


Figure 1: Predicted sludge volumes for primary and secondary wastewater treatment in the United States (16).

or by filling in wastelands. These methods, however, can be quite expensive if the sludge is not concentrated beforehand, since up to 98% of the weight of sludge is water. Therefore, the costs of ultimate disposal can be greatly reduced by decreasing the water content of the sludge and hence reducing the volume and weight of the sludge to be disposed of on land.

In Figure 2, the effect of the different mechanisms used to reduce the water content of the sludge is illustrated. It is apparent from the graph that concentrating by thickening a 1000 gm. sample of a 5% solids content sludge will reduce the weight of the sludge by 67%. Dewatering the sludge would not only reduce the weight further to less than 20% of the original weight, but also concentrate the sludge sufficiently to form a plastic mass. The weight of the sludge may be decreased further by drying but it should be noted that very little volume reduction takes place (dashed line) as the moisture removed by drying will be mainly capillary moisture. Hence, concentration or thickening devices alone will reduce the volume of sludge immensely, however, dewatering is necessary to produce an easily handled and compact cake.

Various dewatering methods have been proposed and utilized to reduce the volume of wastewater sludge before ultimate disposal. These dewatering processes which include vacuum filtration, centrifugation, lagooning, and gravity dewatering on sand beds have all been used with varying success in the past.

The vacuum filter mentioned herein to dewater sludge is a complex mechanical device which consists of a circular drum covered with a

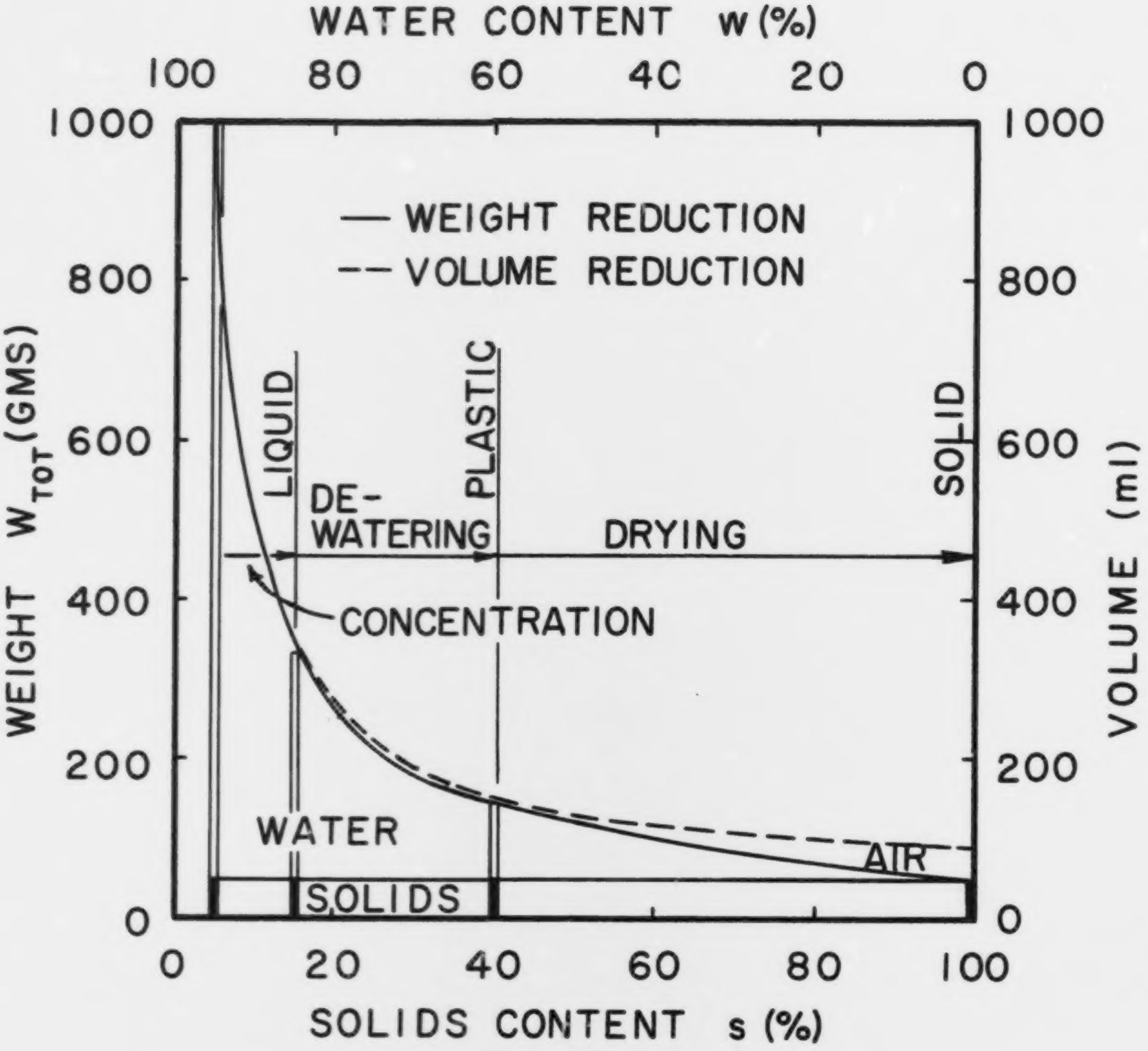


Figure 2: Weight and volume reduction of 1 kg. of wastewater sludge at 5% initial solids content. The water reduction processes are generally effective over the ranges of solids contents shown (17).

filter media. The drum rotates slowly in a trough of incoming sludge, and by application of a vacuum across the filter media, water is separated from the solids, resulting in the formation of a cake. The vacuum is then released, and the sludge cake is scraped off onto a conveyor or is removed by the coil wrap. The sludge must be disposed of after filtration. Heat drying or incineration are common intermediary steps.

The vacuum filter, although efficient at times, has many economic disadvantages. Variable operating conditions frequently require a treatment plant with vacuum filtration to provide a supplemental dewatering process of either sand beds or lagoons or both (3). High maintenance costs are a major disadvantage, as are high chemical and energy costs. The filtrate is often very high in suspended solids and can cause difficulties in recirculation. Skilled operators are demanded for vacuum filters, and these are generally in short supply today, especially in smaller communities.

Sludge lagoons have been used with great success for the dewatering of sludge. The design and operation of a lagoon is similar to that of a sand bed: sludge is applied on natural sand or other highly permeable soil, but it is applied at greater depths than on sand beds. Also, loadings are continuous for a set period of time before the sludge is allowed to dry to a forkable condition. Jeffrey (11) stated that a lagoon, properly run, should be loaded continuously for one year and allowed to dry for eighteen months. After sludge removal, the supporting media is cleaned and "rested" for six months. Therefore, three lagoons would be needed for a continuous dewatering process.

Properly operated, sludge lagoons will dewater sludge of 2 to 6 feet in depth to a final solids content greater than 20% in a year and a half. Drainage will account for the majority of water removed, but evaporation, and in practice, transpiration, are needed to produce a forkable sludge.

Sludge lagoons have the advantage of low maintenance costs since a minimum of personnel is needed and then only when the lagoons are cleaned. However, the initial land cost will be prohibitive for most dewatering operations except for the rural municipality or industry. Either raw or digested sludge may be dispatched, but unless the lagoon is situated away from the populated areas, problems will arise with raw sludge because of esthetics, particularly odors. Flies and mosquitos are additional nuisances.

One of the more recent developments in the dewatering of sludge is centrifugation. This process has been used for many years in the chemical industry to separate liquid from solids. But not until 1959 (2), when the first centrifuge was used with great success on paper mill effluent, did engineers become interested in this process for sludge dewatering. It is reported that centrifuges can concentrate activated sludge from 1.5% solids to 14% solids, and that a mixture of activated sludge and primary sludge could be concentrated to 20% solids. These values are approximately the same for vacuum filtration.

The centrifuge operates continuously, with the sludge fed through a stationary feed pipe and thrown out through feed ports onto a rotating conveyor hub. The solids are settled out on the bowl wall by centrifugal force, and then by means of a helical screw conveyor transported to the end of the machine and discharged.

The functions of centrifuges and vacuum filters are synonymous, as are their limitations, principally the frequent further need for drying or incineration.

After 30 years of research and development of new techniques for dewatering sludge, sand drainage beds still remain the most popular method. In 1965, Quon (19) cited a survey which found that 67% of the municipal sewage plants in the United States were using sand beds, 9.3% were using sludge lagoons, and 6.7% were using mechanical methods. Furthermore, cities with populations of 20,000 or less relied almost exclusively on sand beds.

The design of a sand bed has remained basically the same for some time. The beds are placed preferably on impermeable soil to prevent infiltration to the water table. Clay drainage tiles are placed on the soil and covered with 10-15 cm. of coarse gravel followed by 20-25 cm. of fine gravel and then sand. The beds are enclosed by a concrete wall 30-40 cm. above the sand level. Approximately 20 to 30 cm. of liquid sludge are applied to the bed where gravity dewatering with simultaneous drying by evaporation yields a hard cake of up to 90% solids content. The beds are cleaned prior to refilling, and allow approximately 8-10 applications per year.

CHAPTER II

HISTORICAL DEVELOPMENT AND EARLIER RESEARCH

One of the early researchers on drying beds was Spillner (24). In 1907, he discovered that digested sludge from an Imhoff tank dewatered more rapidly than raw sludge. He found that 80.9% of the moisture reduction in digested sludge was due to drainage as compared with only 44.4% due to drainage for fresh sludge. Spillner concluded that the better drainage of the decomposed sludge was due to a decrease in viscosity, destruction of hydrophilic colloids and organic matter, and the formation of gas bubbles. The gas bubbles and the subsequent lifting of the sludge forming a subnatant was probably the singlemost important factor that increased the drainage rate of the digested sludge. The gases allowed the rapid release of filtrate by unclogging the pores at the sand surface which are otherwise blocked by the sludge floc.

During the 1920's, covering drying beds with greenhouse structures became popular. This increased the rate of drying of the sludge by preventing direct contact with precipitation and maintaining higher temperatures. However, enclosure prevented the flow of air across the beds which removed the water vapor directly above the sludge. In 1931 Skinner (22) published a paper that questioned the economic feasibility of covered sand beds. He stated that open beds had less construction cost per unit area, no maintenance cost for a fragile superstructure, no artificial [illegible]t costs, and no insurance for glass breakage. On the other hand, covered beds needed up to 50% less area per capita, reduced the odor nuisance, and had a better appearance.

Skinner developed one of the first empirical equations for the computation of the area for a drying bed, covered or uncovered.

$$\begin{matrix}\text{Area of Sand Bed}\\ \text{Per Capita}\end{matrix} = \text{Constant}\ \frac{\begin{pmatrix}\text{Average Annual}\\ \text{Precipitation: in.}\end{pmatrix}\begin{pmatrix}\text{Suspended Sewage}\\ \text{Solids: ppm}\end{pmatrix}}{\begin{pmatrix}\text{Number of Months}\\ \text{In Drying Season}\end{pmatrix}\begin{pmatrix}\text{Mean Annual}\\ \text{Temp.: °F.}\end{pmatrix}\begin{pmatrix}\text{Mean Wind}\\ \text{Velocity: mph}\end{pmatrix}}$$

Where: The constant depends on the sewage treatment process. For covered beds, the area calculated above is divided by 2.

The formula points out the different factors that affect sludge drying.

In 1934, Jones (12) made a survey of existing treatment plants to see how effective the sand beds were for the dewatering and drying of sludge. He found the plant records were reliable and concise on every phase of the treatment plant except the sludge dewatering and disposal. As an example:

> "A total of 44 beds of sludge were drawn off from the tanks during this year and a total of 75 beds of scum. All sludge drawn off was well digested, dried out quickly and had no prominent odor."

There was no mention of depth of application, solids content of sludge from digestor, or solids content of sludge when removed from the beds. Jones stated that economical advantages and cost per unit area for sand beds could not be made until more complete records were kept at the treatment plants. With what data he had available, Jones concluded that the degree of treatment, type of process, and character of sludge not only affected the volume of sludge produced, but also the rate of dewatering.

During the same years that Jones was doing research on sand beds, Rudolfs and Heukelekian (21) were studying the drainability of sludge versus the degree of digestion. It was found that well digested sludge dewatered faster than a poorly digested sludge, or a well digested sludge which had been stored before being placed on the sand beds. The resulting conclusion, substantiating the work of Spillner, was that the well digested sludge had a large amount of gas which "buoyed" the sludge solids allowing the subnatant water to drain more rapidly.

For so-called dead sludges (a sludge without entrained or dissolved gases), alum could be added to increase the drainability of the sludge. The alum would react with the carbonates in the sludge forming CO_2 gas which would buoy the sludge solids and increase drainability. Copper sulfate and even dilute sulfuric acid would have the same effect on the sludge. However, addition of alum or any other chemical to a freely gaseous sludge would have no effect on drainability (20). Ferric chloride was tested as a sludge conditioner because of its minimal cost (23). However, the ferric salts were oxidized, clogging the sand pores and allowing less drainage.

It was found in Aurora, Illinois in 1939 (23) that there were differences in drainage characteristics for the same sludge with the same amount of alum, the only difference being the method of adding alum to the sludge. If the alum were completely mixed with prolonged stirring, the sludge would not drain as well as the sludge with a minimum of stirring. This problem was due to a build-up of fines from shearing of the floccules (the same occurs with prolonged pumping). Therefore, in order to minimize the stirring but have complete mixing, alum was dissolved in water first, and then added to the sludge a few feet from the sand bed outlet. Because of the turbulance in the pipe, adequate mixing was assured.

The major disadvantage of sand beds as a sludge dewatering method is the large labor costs to remove the dried sludge from the beds. In small sewage treatment plants the removal costs may be inconsequential being merely an additional occasional task. However, in a larger plant which may treat 2000-3000 cubic yards of sludge every day, the cost of sludge removal becomes very significant. As early as 1931, the Sanitation District of Chicago, aware of the large labor costs to manually remove sludge from drainage beds, constructed a self-propelled sludge removing machine (13). The device was capable of removing 5 cubic yards of sludge per minute and thus replace the 350-400 men which would have been needed to remove equal amounts of sludge. More recently in 1961, a mechanical sludge removing process was installed at Maple Lodge Works in London, England resulting in a substantial decrease in labor costs (14).

One of the first empirical relationships describing the time required for sludge to dewater and dry on sand drainage beds was developed by Haseltine (9) (1). From field data collected at different treatment plants, he plotted the gross bed loading ($kg/m^2/day$) versus solids content of the sludge. The resulting equation was determined

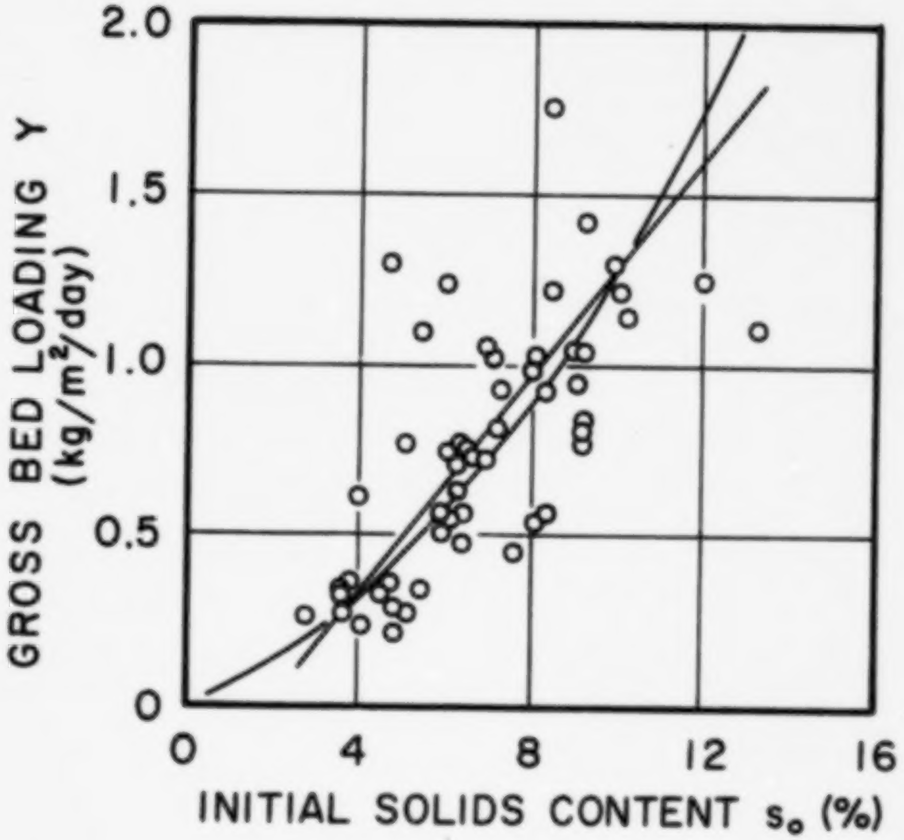


Figure 3: Loading rate of wastewater sludge at various treatment plants in the United States. Shown as a function of the initial solids content, the points and straight line represent the results of Haseltine. The curve is from Vater (1).

to be:

$$Y = 0.157\ S_o - 0.286$$

Y = gross bed loading of solids ($kg/m^2/day$)

S_o = solids content (%)

Vater (1) analyzed the same data used by Haseltine and found that a curve may be fitted using a regression analysis (Figure 3). The relationship thus became exponential and defined by the empirical equation:

$$Y = 0.033\ S_o^{1.6}$$

Y = gross bed loading of solids

S_o = solids content

Although both Haseltine's and Vater's relationships appear to fit the data, the curve drawn by Vater seems more realistic as it goes through the origin where Haseltine's straight line does not. Clearly, a sludge of zero solids content will yield zero bed loading.

Since final moisture content of the sludge was also to be a determining factor as to the sludge dewatering and drying capacity of a sand bed, Haseltine defined the term "net bed loading" as the gross bed loading multiplied by the final water content of the dried sludge. The relationship between net bed loading and solids content was found

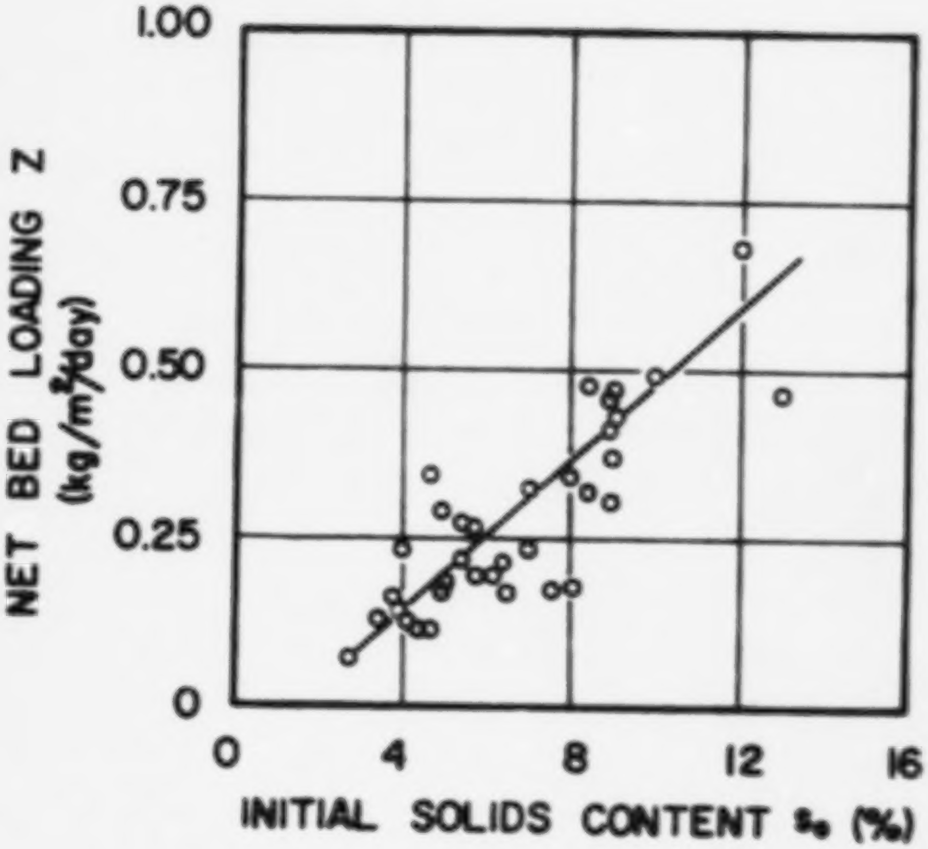


Figure 4: A plot of Haseltine's data of net bed loading versus initial solids content. Data is from American plants. Net bed loading is defined as gross bed loading multiplied by the water content of the sludge removed (9) (1).

to be (Figure 4):

$$Z = 0.057\ S_o - 0.082$$

Z = net bed loading of solids (kg/m^2/day)

S_o = solids content (%)

Although Haseltine's equations are strictly empirical, they have been used for the dimensioning of sludge drying beds. Note that the water content is defined as 100 minus the solids content.

CHAPTER III

RECENT RESEARCH

The previous discussion has primarily dealt with developments in the first half of the twentieth century. During this period there was little progress in formulating dewatering and drying rates to the parameters of the sludge characteristics and quantity, and external factors such as the evaporation potential of the air.

Just as there exists a great difference in dewatering methods, there is also a great difference in types of sludge. However, the differences were related to the treatment process without any other classification except solids content. Since primary and secondary sludges with the same solids content but from different treatment plants will not dewater at the same rate, other drainage characteristics intrinsic to each sludge must be present.

Jeffrey (11) studied the relationship between permeability of the sludge, initial head, and time. For the experiment (see Figure 5), he designed a drainometer which consisted of a lucite tube 18 inches in length covered with a polyethylene film to prevent evaporation. A layer of gravel followed by a 7/8 inch layer of sand was placed in the bottom of the lucite tube to simulate a sand drainage bed. An inverted "U" tube drain with a siphon breaker was inserted between the sand and gravel to maintain a water table. Sludge was poured into the column, after which the volume of subnatant collected and the time were noted. From the experiment, Jeffrey determined a relationship of the form

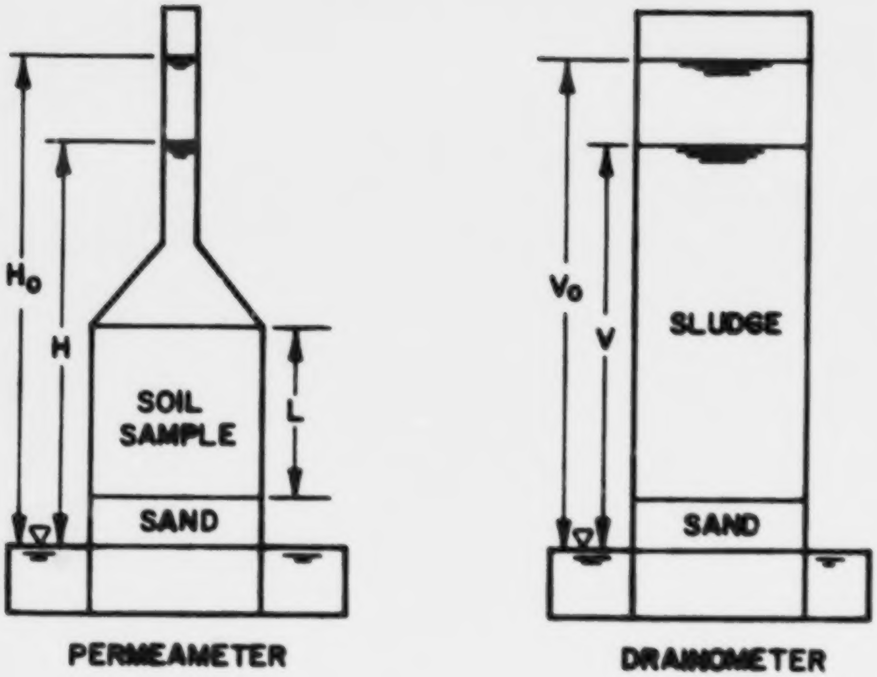

H$_0$
H
L
SOIL SAMPLE
SAND
PERMEAMETER
V$_0$
V
SLUDGE
SAND
DRAINOMETER

Figure 5: A comparison of a typical soil permeameter to Jeffrey's drainometer (11).

$$\log \frac{H}{H_o} = \frac{-Kt}{L}$$

was applicable, which describes flow from a falling head permeameter for soil.

Here: H_o = initial head at $t = 0$

H = head at any time t

K = permeability coefficient of soil sample

L = length of soil sample

t = time

Jeffrey determined that the following equation was valid for sludge dewatering:

$$\log \frac{W_1}{W_o} = \frac{-Kt}{L}$$

Where: W_1 = quantity of filtrate = $V_o - V$ (see Figure 5)

W_o = total quantity of liquor discharged from a drainometer at the time the quantity of discharge over a 24 hour period is equal to 1% of the total

K = permeability coefficient of the sludge

L = depth of sludge

t = time

Jeffrey stated "....the coefficient of permeability, K, is not fixed. Both K and L change with time and apparently at such a rate that the ratio $\frac{K}{L}$ is constant."

In Figure 6 it is readily apparent that Jeffrey's theoretical curves coincide closely to the actual drainage curves for all three different solids contents. However, it should be noted that the theoretical curve could not be drawn until the actual drainage experiments were near completion, since the value of W_o could not be found until the 24 hour drainage in the column was less than 1% of the total filtrate volume.

Jeffrey based his conclusion that the ratio $\frac{K}{L}$ was a constant on his experimental data. The plot of log $\frac{W_1}{W_o}$ versus time approximated a straight line, thus proving that $\frac{K}{L}$ was constant. However, it can be proven that $\frac{K}{L}$ is not a constant but instead proportional to the water content of the sludge (see Appendix for derivation).

$$\frac{K}{L} = B \frac{w^3}{1-w}$$

B = constant

w = moisture content (%)

A plot of w versus $\frac{w^3}{1-w}$ produced a curve (Figure 7). This substantiated the fact that the ratio $\frac{K}{L}$ did not remain constant but instead was a function of water content. Even though one of Jeffrey's conclusions appears, therefore, theoretically unjustifiable, he was the first to develop a relationship between the head and time.

Pilot plant studies on sludge dewatering in England yielded the conclusion that specific resistance (an intrinsic characteristic of

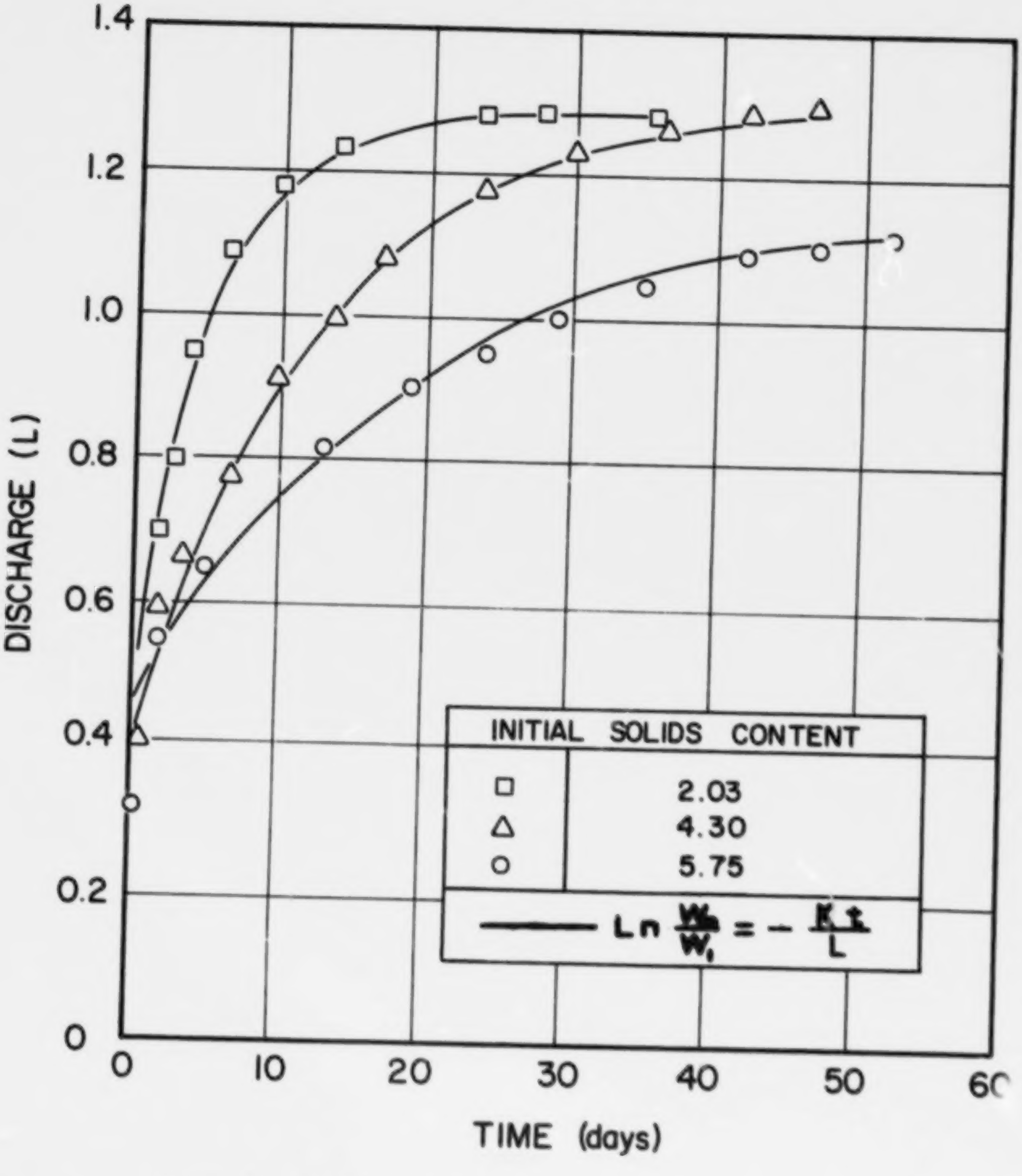


Figure 6: Jeffrey's drainage data. The curves represent Jeffrey's empirically derived equation (11).

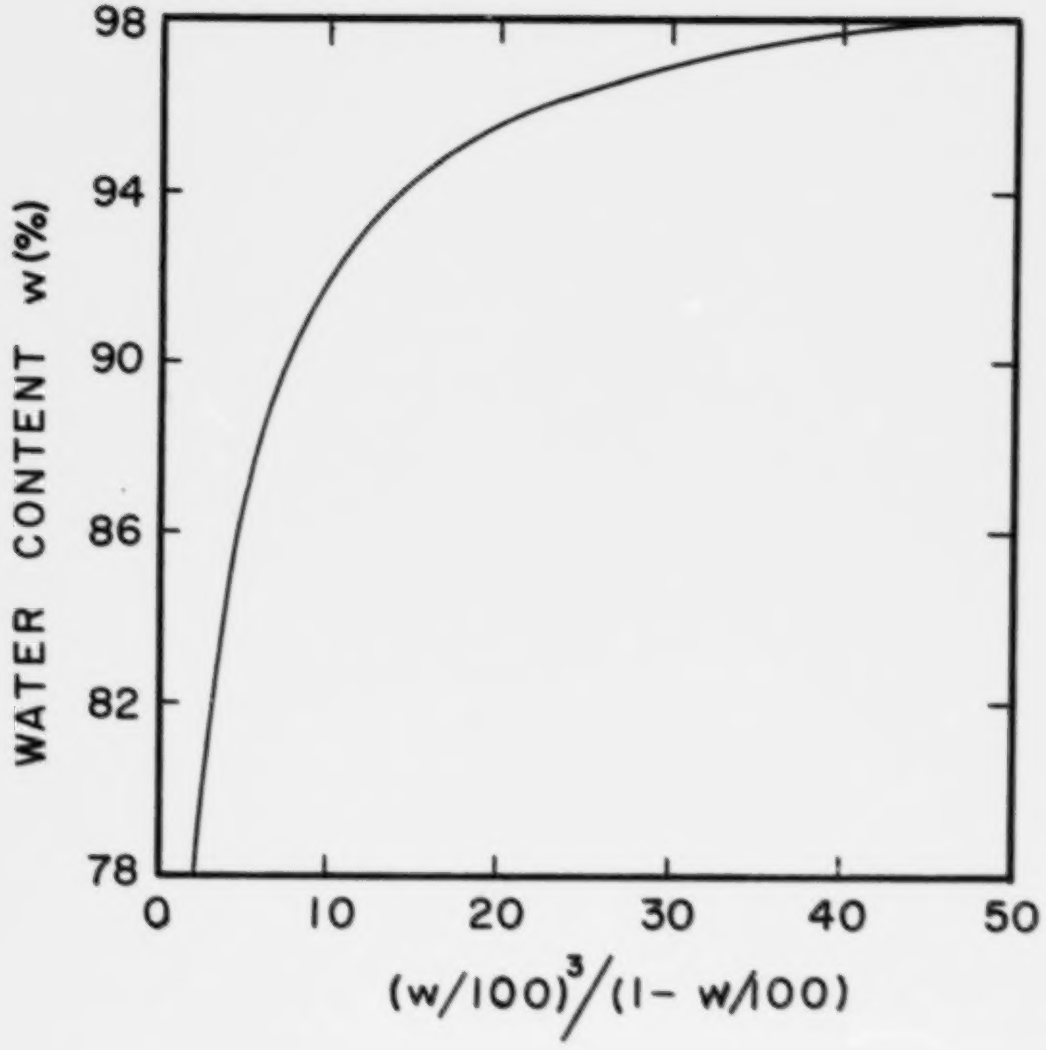


Figure 7: A plot of sludge moisture content w versus $(w/100)^3/(1-w/100)$ to prove that K/L is not constant with a change in moisture content.

sludge which will be discussed in greater detail later) is related to bed loading ($kg/m^2/year$). Specific resistance has long been recognized as a functional parameter to be used to describe the dewatering rates of sludge submitted to vacuum filtration, but this was the first time that specific resistance was tested to be a functional parameter to describe gravity dewatering. This is not unreasonable since the processes of vacuum dewatering and gravity dewatering differ only in type of pressure exerted on the sludge. In vacuum filtration, the vacuum is constant; and in gravity dewatering, the pressure (a function of head) is continually decreasing. A plot of Swanwick's data produced a curve in Figure 8 described by the equation (33) (1):

$$Y' = \frac{10^7}{R^{1/2}}$$

Y' = bed loading ($kg/m^2/year$)

R = specific resistance at 36.9 cm of mercury (sec^2/gm)

Although it is quite possible that there exists a relationship between bed loading and specific resistance, it is highly improbable that this relationship is precisely described by Swanwick's formulation. Outside factors which are related to bed loading and unrelated to specific resistance, such as evaporation and freezing temperatures, most certainly play a considerable role in the dewatering of the sludge. Each value plotted in Figure 8 was an average of three tests run in

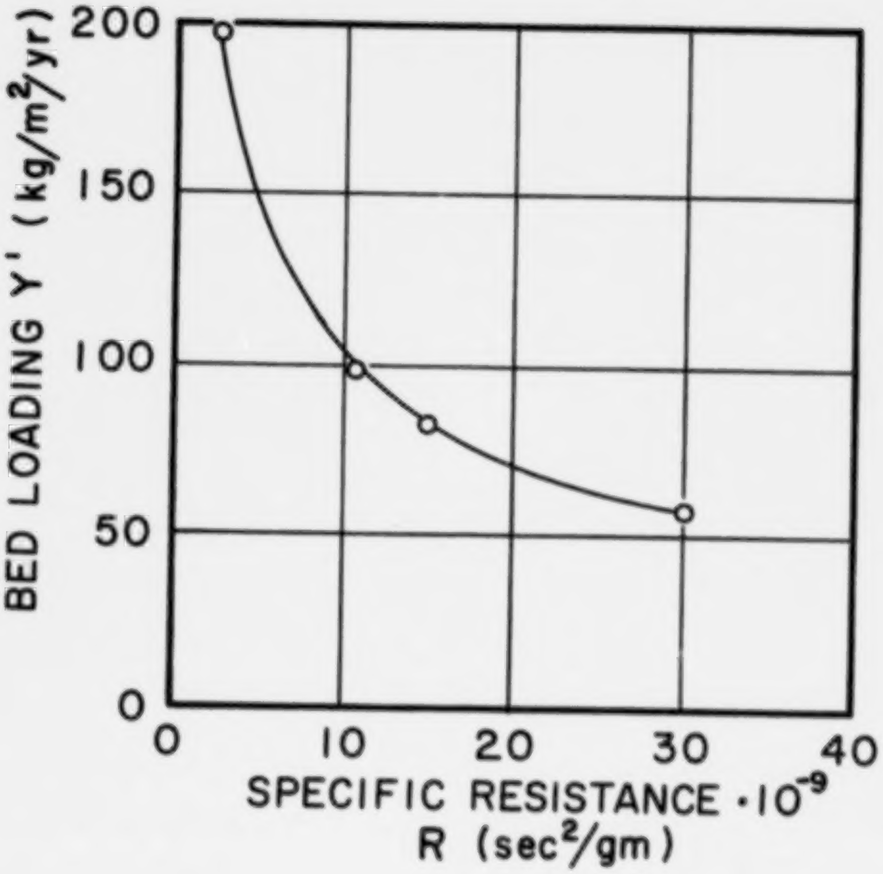


Figure 8: The Swanwick relationship of bed loading to specific resistance. Each point represents the mean of a group of loading values (1) (33).

the course of one year, thus establishing some uncertainty as to the reliability of the relationship.

Specific resistance does not take into account moisture removing mechanisms other than dewatering. Therefore, evaporation and freezing temperatures must be eliminated from scientific studies for proper correlation of specific resistance with bed loading or other dewatering rate parameters.

A few years later, Logsdon and Jeffrey (15) conducted research studying the relationship between the dewatering time of a sludge on sand, and the vacuum dewatering time (a function of specific resistance) of the same sludge using the Buchner funnel test. Sludges with four different solids contents ranging from 2.08%-3.13% total solids were investigated. The times required to dewater equal amounts of water by gravity dewatering and filtration were plotted yielding the curves in Figure 9. Even though the vacuums applied on the four sludges varied from 30.5-61.0 cm. of mercury, Jeffrey concluded that the vacuums in this range dewatered the sludge at equal rates; therefore, Test 3 through Test 6 could be compared. Since these points on the graph appear to approach a straight line asymptotically, Logsdon and Jeffrey concluded that the dewatering relationship was a straight line with the general equation:

$$d = km + i$$

Where: d = number of days drainometer operates

m = number of minutes the vacuum operates

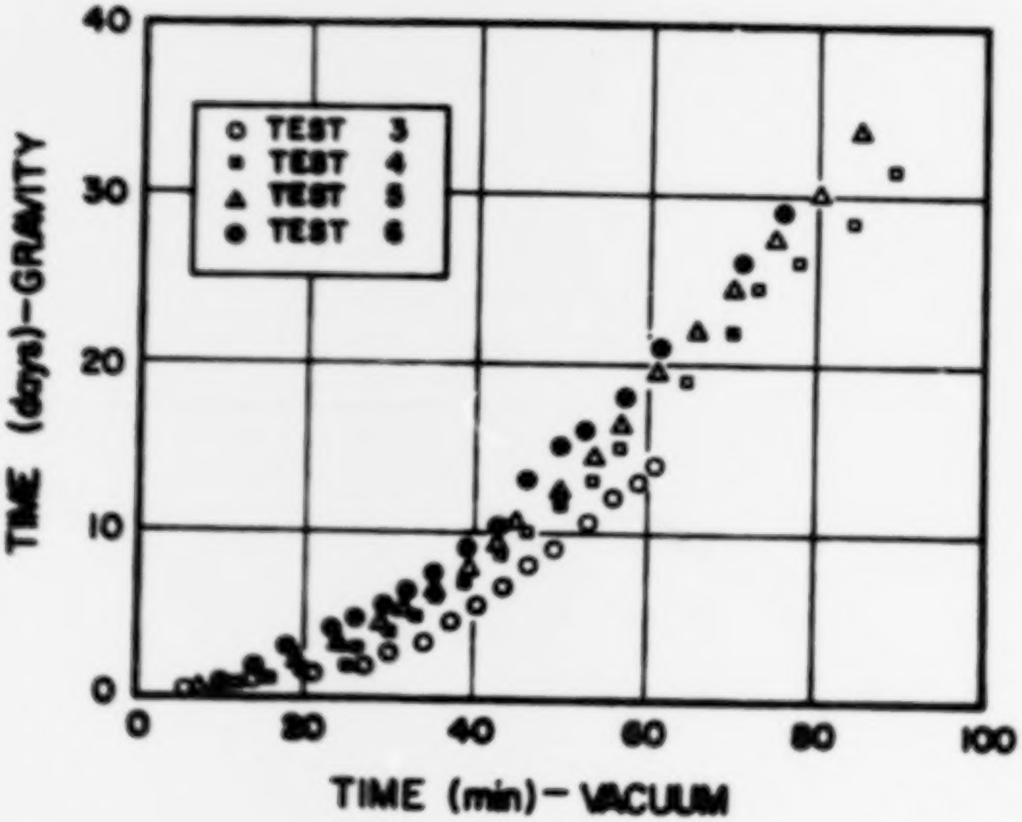


Figure 9: A plot of data accumulated by Logsdon and Jeffrey with dewatering time by gravity versus dewatering time using a vacuum. Solids content of sludge range from 2.08% in Test 6 to 3.13% in Test 3 (15).

k = constant = slope of line

i = intercept of abscissa

Weeks are needed to determine the appropriate constants and unfortunately, each sludge must be re-evaluated by these methods to determine the effect of sludge conditioning.

CHAPTER IV

DERIVATION OF SPECIFIC RESISTANCE

The well known concept of specific resistance initially developed by Carmen and later refined by Coackley and Jones, has been proven to adequately describe the dewatering of a compressible material by means of a vacuum filtration unit. Therefore, since the only difference between dewatering by vacuum filtration and dewatering by gravity is that a constant pressure is applied on the sludge in the former and a continually decreasing pressure is applied on the sludge in the latter, the specific resistance concept could well be the basis for a mathematical model describing gravity dewatering on a sand bed.

The concept of specific resistance can be developed starting with the classic Darcy-Weisbach equation for the head loss of a fluid moving through a pipe.

$$h_f = \frac{\Delta P}{\rho g} = f_1 \left(\frac{L}{D}\right) \frac{\bar{V}^2}{2g} \qquad \text{(Eq. 1)}$$

Where: h_f = head loss (L)

ΔP = pressure drop in pipe ($M\ L^{-1}\ T^{-2}$)

ρ = mass density ($M\ L^{-3}$)

L = length of pipe (L)

$\bar{V}$ = mean velocity ($L\ T^{-1}$)

D = diameter of pipe (L)

g = acceleration due to gravity ($L\ T^{-2}$)

f_1 = friction factor ($M^0\ L^0\ T^0$)

Since the fluid velocity through the pores of a sand bed will be very slow, it is reasonable that a state of laminar flow will exist. The friction factor in the laminar flow region is related to the Reynolds number:

$$f_1 = \frac{64}{IR} \qquad \text{(Eq. 2)}$$

Where: IR = Reynolds Number = $\frac{D\bar{V}\rho}{\mu}$ $(M^0\ L^0\ T^0)$

(Eq. 3)

μ = dynamic viscosity $(M\ L^{-1}\ T^{-1})$

For noncircular conduits the hydraulic radius is defined as:

$$R_h = \frac{A}{P}$$

Where: R_h = hydraulic radius (L)

A = area (L^2)

P = wetted perimeter (L)

Substituting the area and perimeter of a circular cross section

$$R_h = \frac{\frac{\pi D^2}{4}}{\pi D} = \frac{D}{4}$$

$$D = 4R_h \qquad \text{(Eq. 4)}$$

Substituting into (Eq. 1); (Eq. 2), (Eq. 3), (Eq. 4)

$$h_f = f_1\left(\frac{L}{D}\right)\frac{\bar{V}^2}{2g} \qquad \text{(Eq. 1)}$$

$$h_f = \left(\frac{64}{IR}\right)\left(\frac{IR\mu}{DV\rho}\right)\left(\frac{L}{4R_h}\right)\frac{\bar{V}^2}{2g}$$

$$h_f = \frac{2\mu L\bar{V}^2}{\bar{V}R_h{}^2\rho g} \qquad \text{(Eq. 5)}$$

(Eq. 5) is also valid for non-circular pipes.

The derivation thus far has been for the head loss through a cross sectional area of measurable dimensions. For flow through incompressible porous media, the conduit cross section must be expressed in terms of the porosity:

$$A_p = \varepsilon A$$

Where: A_p = cross sectional area of pores available to flow. This area is assumed constant throughout the column of sand (L^2)

ε = porosity = $\dfrac{V_v}{V_{tot}} = \dfrac{\text{Void Volume}}{\text{Total Volume}}$ $(M^0L^0T^0)$

Assume all particles in the porous material to be identical. Also define:

N = number of particles $(M^0L^0T^0)$

S_p = surface area of each particle (L^2)

V_p = volume of each particle (L^3)

Then the hydraulic radius may be written:

$$R_h = \frac{A_p}{P} = \frac{A \cdot L}{P \cdot L} = \frac{V_v}{NS_p} \qquad \text{(Eq. 6)}$$

Note that:

$$\varepsilon = \frac{V_v}{V_{tot}} = \frac{V_v}{V_v + NV_p}$$

$$\varepsilon V_v + \varepsilon N V_p = V_v$$

Hence

$$V_v = \frac{\varepsilon N}{1-\varepsilon}$$

Therefore:

$$R_h = \frac{\varepsilon AL}{NS_p} = \frac{\varepsilon NV_p}{(1-\varepsilon)NS_p} = \frac{\varepsilon V_p}{(1-\varepsilon)\ S_p} \qquad \text{(Eq. 7)}$$

A relationship of the velocity and the superficial velocity $\left(\frac{\text{Total Flow}}{\text{Area of Total Cross-section}} \right)$ must be made. From continuity considerations:

$$V_s A = \bar{V} A_p$$

$$V_s = \frac{\bar{V} A_p}{A} = \frac{\bar{V} A_p L}{AL} = \bar{V}\varepsilon \qquad \text{(Eq. 8)}$$

V_s = superficial velocity ($L\ T^{-1}$)

Combining with (Eq. 7) and (Eq. 8) with (Eq. 5)

$$h_f = \frac{2\mu L \bar{V}^2}{\bar{V} R_h^{\ 2} \rho g} \qquad \text{(Eq. 5)}$$

$$h_f = \frac{(2\mu L)\left(\frac{V_s}{\varepsilon}\right)(1-\varepsilon)^2\ S_p^{\ 2}}{\rho g \qquad \varepsilon^2 \bar{V}_p^{\ 2}} = \frac{2(1-\varepsilon)^2\ V_s \mu L S_p^{\ 2}}{\varepsilon^3 g V_p^{\ 2} \rho} = \frac{\Delta P}{\rho g} \qquad \text{(Eq. 9)}$$

Set

$$\frac{2(1-\varepsilon)^2\ S_p^{\ 2}}{\varepsilon^3\ V_p^{\ 2}} = R \qquad \text{(Eq. 10)}$$

Where R = media or filter resistance (L^{-2})

$$\frac{h_f \rho g}{L} = \frac{\Delta P}{L} = \mu V_s R \qquad \text{(Eq. 12)}$$

Solving for V_s

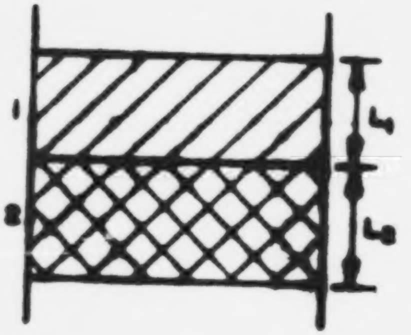
$$V_s = \frac{\Delta P}{L\mu R} = \frac{1}{A}\frac{dV}{dt}$$

$$\frac{dV}{dt} = \text{Flowrate } (L^3\ T^{-1})$$

Therefore:

$$\frac{dV}{dt} = \frac{A\Delta P}{\mu LR} \qquad \text{(Eq. 13)}$$

For stratified filters where there are two different depths (L) and two different resistances, (Eq. 13) may be used.



FILTER (1)

$$\left(\frac{dV}{dt}\right)_1 = \frac{A_1\Delta P_1}{\mu_1 L_1 R_1}$$

FILTER (2)

$$\left(\frac{dV}{dt}\right)_2 = \frac{A_2\Delta P_2}{\mu_2 L_2 R_2}$$

Since water is incompressible

$$\left(\frac{dV}{dt}\right)_1 = \left(\frac{dV}{dt}\right)_2 = \frac{dV}{dt}$$

$$A_1 = A_2 = A$$

$$\mu_1 = \mu_2 = \mu$$

$$\frac{A\Delta P}{\mu LR} = \frac{A\Delta P_1}{\mu L_1 R_1} = \frac{A\Delta P_2}{\mu L_2 R_2}$$

$$\frac{\Delta P}{LR} = \frac{\Delta P_1}{L_1R_1} = \frac{\Delta P_2}{L_2R_2}$$

$$\Delta P_2 = \frac{L_2R_2}{L_1R_1} \quad \Delta P_1$$

With
$$\Delta P = \Delta P_1 + \Delta P_2$$

$$L = L_1 + L_2$$

$$\Delta P = \Delta P_1 + \frac{L_2R_2}{L_1R_1} \quad \Delta P_1$$

If the filter consists of n number of strata

$$\frac{L_1R_1}{\Delta P_1} = \frac{L_2R_2}{\Delta P_2} = \frac{LR}{\Delta P}$$

$$LR = \frac{\Delta P}{\Delta P_1} L_1R_1 = (\Delta P_1 + \frac{L_2R_2}{L_1R_1} \Delta P_1) \frac{L_1R_1}{\Delta P_1}$$

$$LR = L_1R_1 + L_2R_2 + \sum_{n=3}^{n=\infty} L_n \cdot R_n \qquad \text{(Eq. 14)}$$

Combining (Eq. 13) and (Eq. 14)

$$\frac{dV}{dt} = \frac{A\Delta P}{\mu LR} = \frac{A\Delta P}{\mu(L_1R_1+L_2R_2)} = \frac{A\Delta P}{\mu \sum\limits_{n=1}^{\infty} L_nR_n} \qquad \text{(Eq. 15)}$$

In 1933, Carmen (5) developed the specific resistance concept which described the filtration characteristics of a compressible material. The concept was developed specifically for determining the filter cake yield from a vacuum filtration system. However, not until 1956 when Coackley standardized the method for determining the specific resistance did the full potential of this concept become available.

Filtration of a compressible material on a porous medium presents a unique solution. Initial filtration is dependent on the support medium; and gradually a cake forms on the support surface. This cake increases in thickness with increasing volume of filtrate; as each volume of filtrate is separated from the sludge, a portion of this volume (solids) will remain to form a sludge cake with a thickness equal to L (Figure 10). The relationship is expressed as

$$L_1 = \frac{vV}{A}$$

Where: V = volume of filtrate (L^3)

A = area of surface (L^2)

v = volume of cake deposited per unit volume of filtrate ($L^0\ M^0\ T^0$)

This accumulation of sludge particles, however, may be more easily expressed and determined if v is replaced by f, the weight of cake solids per unit volume of filtrate ($F\ L^{-3}$). Therefore, substitution into (Eq. 15) will give:

$$\frac{dV}{dt} = \frac{A\Delta P}{\mu(\frac{fVR}{A} + L_2R_2)}$$

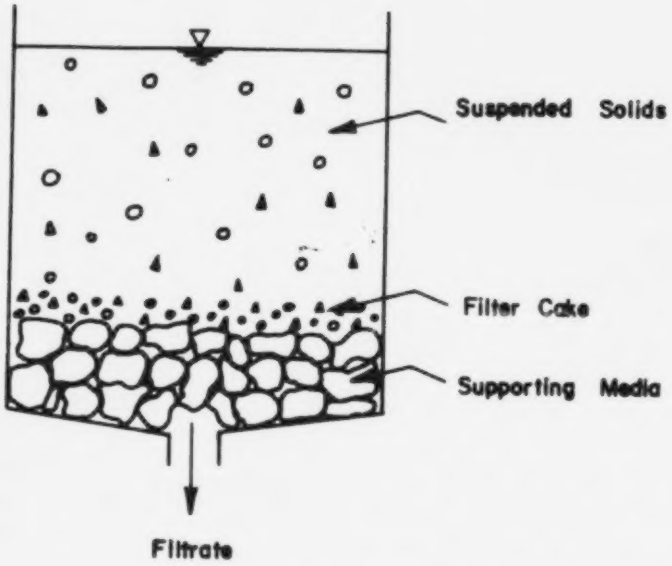

Suspended Solids
Filter Cake
Supporting Media
Filtrate

Figure 10: A schematic representation of filtration of a compressible material on a porous medium. As the filtrate drains from the sludge, solids are left behind at the sludge- filter interface forming a filter cake. The specific gravity of sludge particles is about 1.2, thus the sludge particles, mostly of micron size, will settle very slowly, justifying the assumption that the rate of buildup of the filter cake is directly proportional to the volume of filtrate only, and not to the settling velocity of the particles.

L_2 and R_2 refer to the supporting medium. The resistance of the cake, R, called "specific resistance," has units of T^2M^{-1}.

The characteristics of the supporting incompressible medium may be combined by substituting R_f for the constant L_2R_2 and rearranging:

$$dt = \left(\frac{\mu fVR + \mu AR_f}{A^2\Delta P}\right) dV$$

If ΔP remains constant, the following integration can be made:

$$\int_0^t dt = \int_0^V \left(\frac{\mu fVR}{A^2\Delta P} + \frac{\mu AR_f}{A^2\Delta P}\right) dV$$

$$t = \frac{1}{2}\frac{\mu fR}{A^2\Delta P}V^2 + \frac{\mu R_f}{A\Delta P}V \qquad \text{(Eq. 16)}$$

With the experimental data, plotted as t divided by V, $(\frac{t}{V})$ versus V will yield a straight line on arithmetic paper. The slope b allows calculation of the specific resistance R (Figure 11)

$$R = \frac{2A^2\Delta Pb}{\mu f} \qquad \text{(Eq. 17)}$$

Where:

R = specific resistance of compressible material submitted to a vacuum of ΔP (T^2M^{-1})

ΔP = pressure loss across sludge cake ($F\ L^{-2}$)

A = area of filtration surface (L^2)

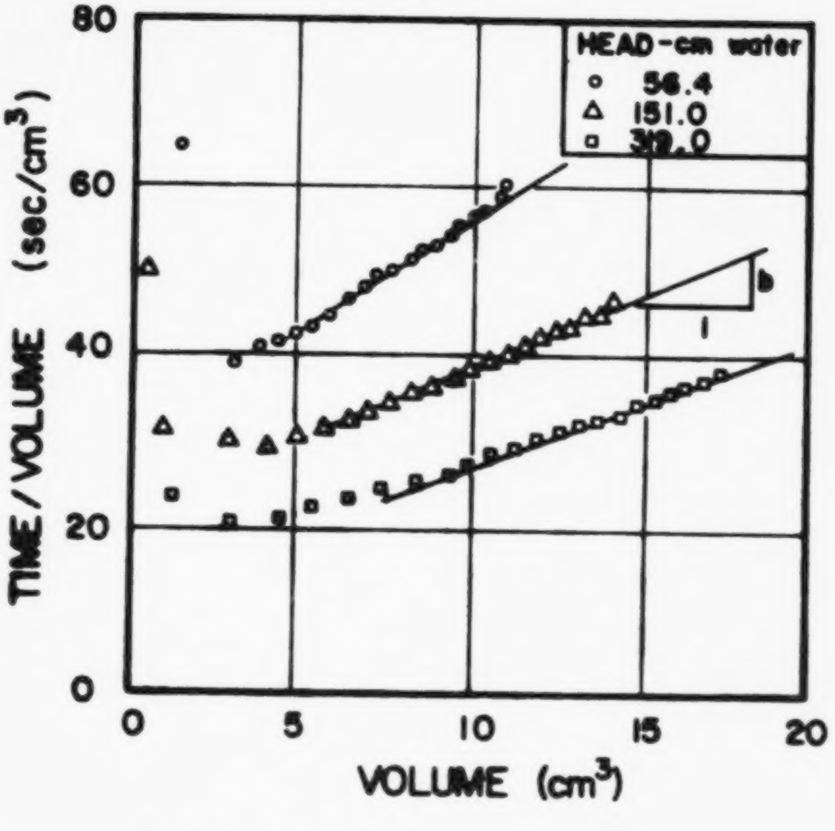


Figure 11: A typical graph used to determine the specific resistance of a wastewater sludge at three different vacuums. From Experiment IV.

μ = dynamic viscosity of filtrate ($M\ L^{-1}\ T^{-1}$)

f = weight of solids per unit volume of filtrate ($F\ L^{-3}$)

b = slope from $\frac{t}{V}$ versus V plot ($T\ L^{-6}$)

Similarly, the $\frac{t}{V}$ intercept of the line of slope b at V = 0 determines the resistance of the supporting media

$$R_f = \frac{(\frac{t}{V}\text{ intercept})\ A\Delta P}{\mu}$$

It is apparent from (Eq. 17) that the specific resistance of a compressible material is dependent on pressure. This pressure dependency will offer no problem if the test vacuum applied to the sludge to determine the specific resistance is the same as the vacuum applied to the sludge by the vacuum filter. However, to enable the calculation of filtration yields at different vacuums without running more specific resistance tests, the relationship of specific resistance to pressure was studied and found to be empirically described by the following equation (35), (36):

$$R = C(\Delta P)^{\sigma} \qquad \text{(Eq. 18a)}$$

Where:

R = specific resistance of sludge submitted to a vacuum of ΔP ($T^2\ M^{-1}$)

C = cake constant = specific resistance when $\sigma = 0$ ($T^2\ M^{-1}$)

σ = coefficient of compressibility ($L^0\ M^0\ T^0$)

ΔP = pressure loss across cake associated with R

A plot of the log of specific resistance as the ordinate versus the log of its respective pressure loss will yield a straight line with a slope equal to the coefficient of compressibility σ. The specific resistance for any pressure loss may thus be determined with values for the coefficient of compressibility and cake constant C.

CHAPTER V

DERIVATION OF MATHEMATICAL MODEL

Using the specific resistance concept, a mathematical model can be derived for determining the gravity dewatering rate of a wastewater sludge or other compressible material on a drainage bed.

It has been previously mentioned that the specific resistance of a compressible material is proportional to the pressure loss across a filter cake (35) (36).

$$R = C(\Delta P)^{\sigma} \qquad \text{(Eq. 18a)}$$

In order for the above equation to be dimensionally correct ΔP must be dimensionless. This is, of course, a fallacy since ΔP will have units of force per unit area. Therefore, the equation is rewritten yielding:

$$R = R_c\left(\frac{\Delta P}{\Delta P_c}\right)^{\sigma} \qquad \text{(Eq. 18b)}$$

Where: R_c = specific resistance at vacuum ΔP_c

ΔP_c = the vacuum associated with a specific resistance value of R_c.

ΔP may refer to either a vacuum or a pressure. It is proportional in gravity dewatering to the instantaneous head of the sludge on a sand bed. This relationship is described by the formula:

$$\Delta P = \rho h g$$

Where:

ΔP = pressure loss across sludge cake ($M\ L^{-1}\ T^{-2}$), ($F\ L^{-2}$)

ρ = mass density of sludge ($M\ L^{-3}$)

h = head acting on sludge at any time (L)

g = acceleration due to gravity ($L\ T^{-2}$)

Therefore:

$$R = R_c\left(\frac{\rho hg}{\rho h_c g}\right)^{\sigma} = R_c\left(\frac{h}{h_c}\right)^{\sigma} \qquad \text{(Eq. 19)}$$

h_c = head loss associated with a specific resistance value of R_c.

Referring back to (Eq. 15)

$$\frac{dV}{dt} = \frac{A\Delta P}{\mu(L_1R_1 + L_2R_2)} \qquad \text{(Eq. 15)}$$

And knowing that:

$$L_2R_2 = R_f$$

$$L_1 = \frac{fV}{A}$$

Substitution of R on (Eq. 19) for R_1 in (Eq. 15) and rearranging gives:

$$\frac{dV}{dt} = \frac{A^2\rho hg}{\mu(fVR_c\left(\frac{h}{h_c}\right)^{\sigma} + AR_f}$$

Because the specific resistance of a wastewater sludge is much greater than the resistance of the supporting media, sand or soil: $R >> R_f$.

The term AR_f can be assumed negligible. Therefore it follows

$$\frac{dV}{dt} = \frac{A^2 \rho h g}{\mu f V R_c \left(\frac{h}{h_c}\right)^{\sigma}}$$

Since:

$$\frac{dV}{dt} = -A\frac{dh}{dt}$$

And

$$V = A(H_o - h)$$

H_o = initial head (L)

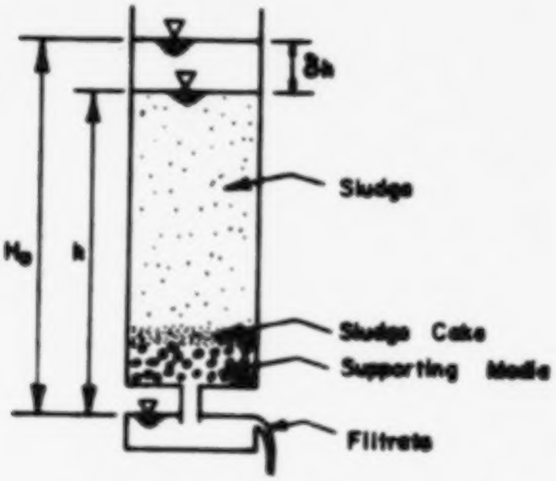

δh
Sludge
H_o
h
Sludge Cake
Supporting Media
Filtrate

Figure 12: Schematic representation of the different terms used in the derivation of the mathematical model.

Therefore:

$$\frac{dh}{dt} = -\frac{\rho g h \left(\frac{h_c}{h}\right)^{\sigma}}{\mu f R_c (H_o - h)}$$

Reorganizing and integrating to the proper limits will yield

$$\int_0^t dt = -\frac{R_c \, \mu f}{\rho g (h_c)^{\sigma}} \int_{H_o}^{h} (H_o - h) h^{\sigma - 1} dh$$

$$t = -\frac{\mu R_c f}{\rho g (h_c)^\sigma}\left[\left[\frac{H_o h^\sigma}{\sigma}\right]_{H_o}^{h} - \left[\frac{h^{\sigma+1}}{\sigma+1}\right]_{H_o}^{h}\right]$$

$$t = -\frac{\mu R_c f}{\rho g (h_c)^\sigma}\left[\frac{H_o}{\sigma}(h^\sigma - H_o{}^\sigma) - \frac{h^{\sigma+1} - H_o{}^{\sigma+1}}{\sigma+1}\right]$$

$$t = \frac{\mu R_c f}{\rho g (h_c)^\sigma (\sigma+1)}\left[h^{\sigma+1} - \frac{\sigma+1}{\sigma} H_o h^\sigma - H_o{}^{\sigma+1} + \frac{\sigma+1}{\sigma} H_o{}^{\sigma+1}\right]$$

(Eq. 20)

The value of f, solids deposited per unit volume of filtrate, requires further analysis in order to simplify Equation 20.

If no solids are lost during filtration the volume of filtrate is

$$V = \frac{(W_{tot_o} - W_{tot_f})}{\rho g}$$

ρ = mass density of filtrate ($M\ L^{-3}$)

g = acceleration due to gravity ($L\ T^{-2}$)

W_{tot_o} = weight of unfiltered sludge ($M\ L\ T^{-2}$)

W_{tot_f} = weight of filtered cake ($M\ L\ T^{-2}$)

Referring to the total weight in terms of solids content:

$$W_{tot} = \frac{W_{ts}}{F_o}(100)$$

W_{tot} = total weight of sludge ($M\ L\ T^{-2}$)

W_{ts} = weight of solids in unfiltered sludge ($M\ L\ T^{-2}$)

W_{ts} = weight of solids in filtered cake ($M\ L\ T^{-2}$)

F_o = solids content (%)

Therefore the volume of filtrate may be written as:

$$V = \frac{\frac{W_{ts} \cdot 100}{F_o} - \frac{W_{ts} \cdot 100}{F_f}}{\rho g}$$

F_o = solids content of sludge at beginning of test (%)

F_f = solids content of sludge at end of test (%)

which simplifies to

$$V = 100 \frac{W_{ts}}{\rho g} \left(\frac{1}{F_o} - \frac{1}{F_f} \right) \qquad \text{(Eq. 21)}$$

It has been previously defined that f is equal to the weight of dry solids deposited per unit volume of filtrate.

$$f = \frac{W_{ts}}{V} \qquad \text{(Eq. 22)}$$

Therefore, by substitution of (Eq. 21) into (Eq. 22)

$$f = \frac{W_{ts}}{\frac{100W_{ts}}{\rho g} \left(\frac{1}{F_o} - \frac{1}{F_f} \right)}$$

which reduces to

$$f = \frac{\rho g}{\frac{100}{F_o} - \frac{100}{F_f}} \qquad \text{(Eq. 23)}$$

F_o will be much less than F_f since dewatering will increase the solids content of a dilute sludge 10 - 15 times. Therefore, the term $\frac{100}{F_f}$ may be neglected. Dropping the subscript:

$$f = \frac{\rho g F}{100} \qquad \text{(Eq. 24)}$$

Equation 24 may now be substituted into Equation 20, resulting in

$$t = \frac{\mu R_c F}{100(h_c)^{\sigma}(\sigma+1)} \left(h^{\sigma+1} - \frac{\sigma+1}{\sigma} H_o h^{\sigma} - H_o^{\sigma+1} + \frac{\sigma+1}{\sigma} H_o^{\sigma+1} \right)$$

(Eq. 25)

Equation 25 is dimensionally correct with the gravity dewatering time defined by basic parameters -- specific resistance R_c at a given head loss h_c, coefficient of compressibility σ, initial solids content F, initial head H_o, and dynamic viscosity of the filtrate μ.

CHAPTER VI

EXPERIMENTAL DETERMINATION OF SPECIFIC RESISTANCE AND COEFFICIENT OF COMPRESSIBILITY

Since specific resistance was the basic characteristic of sludge to be used in the formulation of a mathematical model describing drainage properties, great care was taken to devise an accurate method of analysis. However, it was determined by a few experiments that a simple setup (Figure 13) of a vacuum pump, a mercury manometer, a 50 ml. burette, a vacuum reservoir, and a porcelain Buchner funnel (12 cm. I.D.) gave consistent results.

The procedure used to accumulate the necessary data for determining the specific resistance in the Buchner funnel test was as follows:

1. Place one Whatman No. 5 filter paper on the bottom of the Buchner funnel.
2. Wet paper with distilled water.
3. Apply a vacuum of 10 inches of mercury for approximately ten seconds to remove excess water.
4. Measure out 100 ml. of sludge of predetermined solids content and pour into Buchner funnel.
5. Apply vacuum desired for 660 seconds and record filtrate volume every 30 seconds.
6. Repeat steps 1-5 for two other vacuums in the range of 2 to 15 inches of mercury.

The specific resistance of each sludge in the six experiments run was found by the method proposed by Coackley (5). A plot was made of

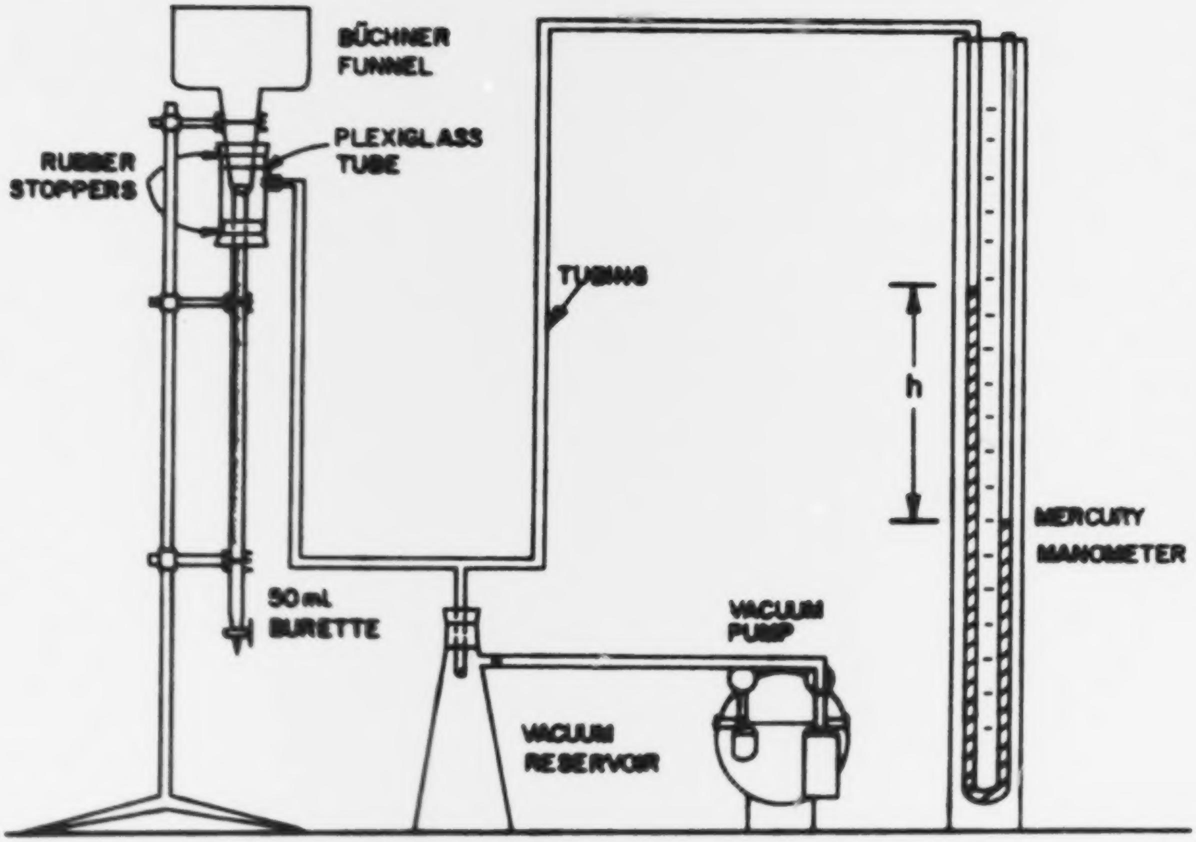

BÜCHNER FUNNEL
PLEXIGLASS TUBE
RUBBER STOPPERS
TUBING
h
MERCURY MANOMETER
50 ml BURETTE
VACUUM PUMP
VACUUM RESERVOIR

Figure 13: Apparatus to determine the specific resistance of compressible sludges.

time/volume of filtrate, versus volume of filtrate; the resulting slope being equal to

$$b = \frac{Rf\mu}{2\Delta PA^2} \qquad \text{(Eq. 26)}$$

Where: μ = viscosity of filtrate ($ML^{-1}T^{-1}$)

b = slope $\frac{t}{V}$ versus V plot (TL^{-6})

ΔP = vacuum across filter cake = $S_{hg}\rho h_{hg}$ (FL^{-2})

S_{hg} = specific gravity of mercury ($M^0L^0T^0$)

h_{hg} = height of mercury in manometer (L)

ρ = mass density of water (ML^{-3})

g = acceleration due to gravity (LT^{-2})

A = area of Buchner funnel (L^2)

f = weight of sludge per unit volume of filtrate

$= \frac{\rho gF}{100}$ (FL^{-3})

F = initial solids content of sludge (%)

R = specific resistance (T^2M^{-1})

The viscosity of the filtrate was assumed to be that of water at room temperature (35) after separate initial experiments had shown this to be true. The vacuum across the cake, constant throughout each run, was measured from the mercury manometer. The filter area is very important since it is a squared term in (Eq. 26). Swanwick (25) said that if the area of the bottom of the Buchner funnel were to be used, a wire gauze should be placed between the perforated part of the funnel and the filter paper, so that the entire filter paper would be exposed to the vacuum. However, it was proven that insertion of the gauze had no appreciable affect on the drainage rate (31). Therefore, the area of the Buchner funnel was used in all calculations; 113 sq. cm. The solids content f is the total solids per unit volume of filtrate. It was determined per Standard Methods (37).

The slope of the time/volume versus volume curve was calculated by two different methods (two point and regression analysis) and checked by plotting to prove that a linear relationship existed. After much experimentation, it was decided to run the specific resistance test for 660 seconds providing enough data, 22 points, to correctly estimate the slope. The curve, however, was not linear through the first four points. (This was due to the absence of a sludge cake thick enough to restrict the flow of filtrate.) Therefore, the sixth and twenty-second points were used to calculate the slope by the two point method. The resulting slope in every experiment was substantially lower than the slope determined by regression analysis using points six through twenty-two. Since the regression analysis would yield a slope closer to the absolute slope of the line, it was used to calculate the specific resistance of each wastewater sludge.

Once the specific resistances of a particular wastewater sludge was calculated at three different vacuums, the coefficient of compressibility could be determined. Using (Eq. 18b) which defines the relationship of the coefficient of compressibility to specific resistance and its respective head loss, the coefficient of compressibility can be calculated.

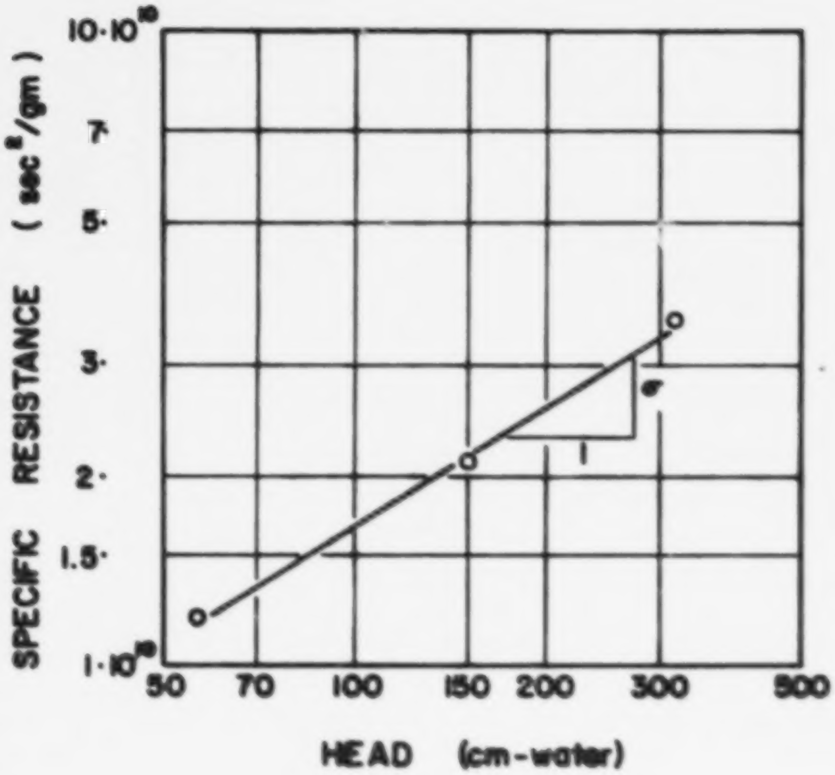


Figure 14: A plot of specific resistance versus head (converted from inches of Hg to cm of water) with a resulting slope equal to coefficient of compressibility. From Experiment IV.

$$R = R_c \left(\frac{\Delta P}{\Delta P_c}\right)^{\sigma} \qquad \text{(Eq. 18b)}$$

Where: R = specific resistance at vacuum ΔP (T^2M^{-1})

ΔP = vacuum loss across cake (inches of Hg)

ΔP_c = vacuum loss across cake (inches of Hg)

R_c = specific resistance at vacuum ΔP_c (T^2M^{-1})

σ = coefficient of compressibility

A bi-logarithmic plot (Figure 14) of the specific resistances versus their respective vacuums yields a straight line with a slope equal to the coefficient of compressibility. The value of R_c is arbitrary, depending on the value of ΔP_c selected. Generally R_c was read as the specific resistance from the line of best fit at ΔP_c = 2.5 cm of Hg. Thus R_c could then be defined as the specific resistance of the sludge if it were not compressible (σ=0). However, as stated previously, ΔP_c may be any vacuum.

CHAPTER VII

EXPERIMENTAL DETERMINATION OF GRAVITY DEWATERING RATES

In order to obtain statistically significant and accurate results, nine columns (Figure 15 and Figure 16) were fabricated for the research project. The column diameter, not a determining factor in the results as pointed out by previous investigators (20), was 4.0 inches (10.2 cm) I.D., large enough to insert a hand but small enough so that only a minimal amount of sludge would be needed for the experiment. The corrosive-resistant acrylic columns were 1/4 inch (0.635 cm) thick and 36 inches (91.5 cm) long. At each end of the columns, flanges were fastened so that acrylic plates could be bolted in place with an O-ring seal to prevent leakage of water and air. Nickel-plated nozzles, attached to the plates connected at each end of the column, were the only outlets. O-ring seals were also used to prevent leakage from the nozzle connection.

Placed directly below each column were 1000 ml lucite cylinders, graduated to 2 milliliters. A No. 12 rubber stopper with two holes was placed in each graduated cylinder with 1/4 inch (0.625 cm) I.D. Tygon tubing connected to a glass tube in the stopper and to the lower nozzle of the column. Tygon tubing was also connected to the top nozzle of the column which ran to another glass tube in the stopper, thus preventing evaporation and maintaining a saturated humidity. No pressure differences could exist in the column and the cylinder, for the filtrate displaced the same volume of air in the graduated cylinder as would be needed by the volume vacated by the filtrate in the column.

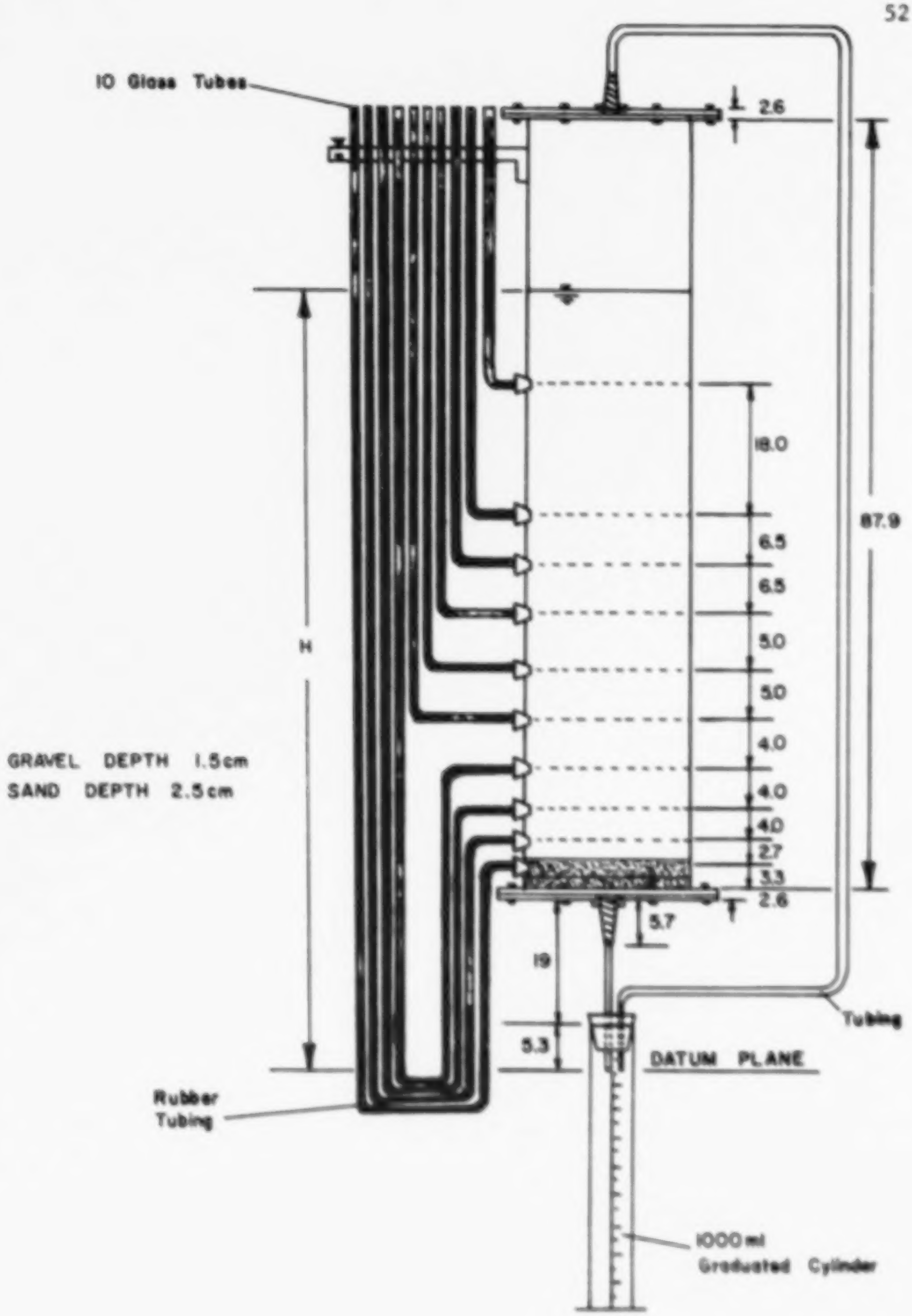

10 Glass Tubes
2.6
87.9
18.0
6.5
6.5
5.0
5.0
4.0
4.0
4.0
2.7
3.3
2.6
H
GRAVEL DEPTH 1.5cm
SAND DEPTH 2.5cm
5.7
19
5.3
DATUM PLANE
Tubing
Rubber Tubing
1000 ml Graduated Cylinder

Figure 15: Gravity dewatering apparatus with piezometer tubes. The supporting filter media consists of 2.5 cm of sand underlain by approximately 1.5 cm of coarse gravel. The top sand surface lies 30.9 cm above the datum plane. All dimensions are in cm.

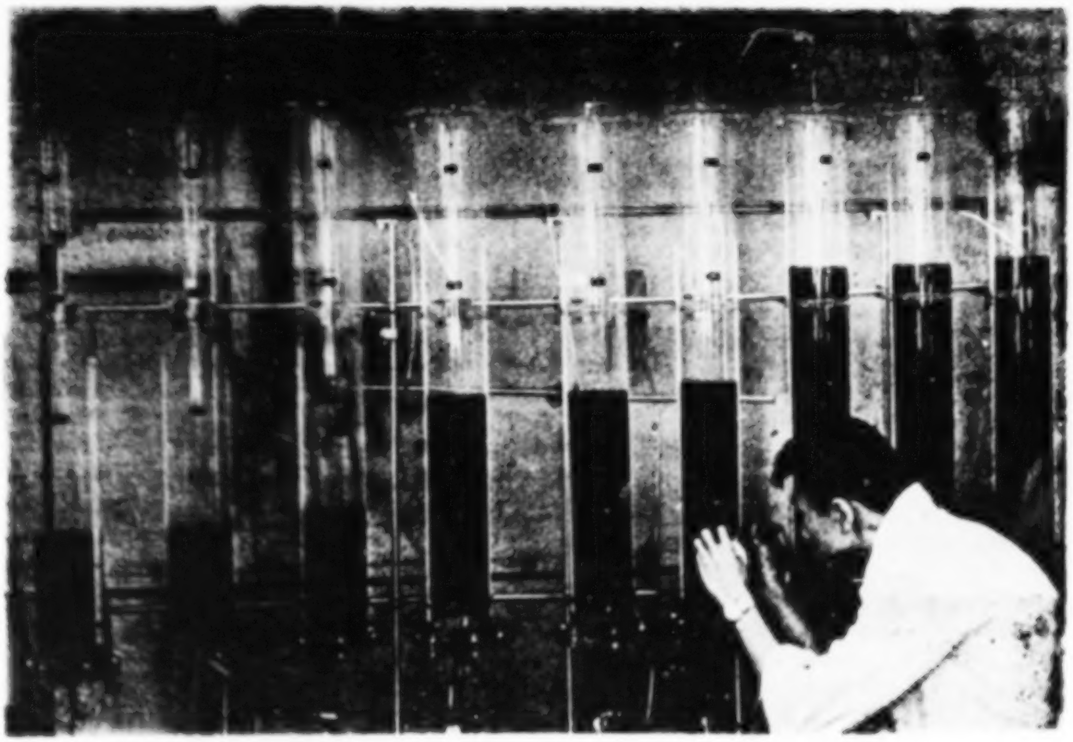


Figure 16: Gravity dewatering columns

The choice of nine columns is dictated by statistical considerations to allow simultaneous experiments with two variables. Here (Experiment 5) the variables include three types of sand and three different filling depths (11, 41, and 81 cm). A sludge solids-supernatant interface is being observed.

In order to prove that the drainage rate of a sludge on a sand bed was a function of specific resistance, coefficient of compressibility, initial solids content, and initial head; interference from outside factors which would affect the rate of moisture removal but which was unrelated to gravity drainage had to be eliminated. These factors included evaporation and flotation.

Evaporation, if not eliminated, would reduce the moisture from the sludge at some unknown rate so that determination of the head from the volume of filtrate collected would be impossible. Therefore, the experimental apparatus was designed as a closed system preventing the entrance or exit of moisture.

Flotation, which is the lifting or "buoying" of the sludge solids from the supporting media, could have a pronouced affect on drainage rates due to a rapid release of subnatant. Flotation can occur in sludges by the attachment of gas bubbles to the sludge particles causing upward movements. These gas bubbles form as a result of continuing fermentation, or subjection of the sludge to lower pressures, such as found within the sand-gravel region of the columns. To curtail flotation, the sludge was autoclaved at 110°C. for 5 minutes, driving dissolved gases from solution as well as killing all gas-producing bacteria. The specific resistance of the sludge is disturbed by autoclaving (10), but as this factor was measured on the cooled sludge, the same sludge as used in the columns, experimental verification of the theory was not affected.

All quantitative determinations such as solids content, grease content, and pH were made on autoclaved sludges, and followed the appropriate procedures outlined in the 12th edition of Standard Methods for the Examination of Water and Wastewater.

Each column was initially calibrated with water by noting the time required for the water surface elevation to drop from 110 cm to 30 cm. Minor adjustments were made until the time deviation between columns was 1.9 per cent. The addition of the sand, gravel, and the chromel 16 mesh (24 B&S gage) wire screen decreased the discharge rates in each column equally. Total drainage time using water was approximately 2 minutes. Hence, the permeability of the supporting media was so large relative to the permeability of the sludge that it could be disregarded in the computations.

It is important to note that the effective dewatering head was the distance from the top sludge surface to the tip of the discharge tube located in the graduated cylinder. The distance from the discharge tube to the sand surface approximated the standard design depths for sand in a drying bed. The additional head created is thought to realistically approach the operation of properly constructed drying beds, for the sand becomes rapidly saturated tending to place a vacuum on the bottom of the sludge-sand interface, as was created by the discharge system of the columns. This assumption serves merely to accelerate dewatering, and is not critical for experimental verification.

Three different sands were utilized for the experiments. To provide a basis for later corroboration of the experiments by other researchers, A.S.T.M. Standard Ottawa sand was used. Also to provide information of practical value, sands from drying beds of two nearby treatment plants (Hermitage Hills, and Franklin, Tennessee) were tested. The sand types are coded as Type O, H, and F, and were sieve analyzed.

Before each of the six experiments, all equipment including the acrylic columns, graduated cylinders, and Tygon tubing was thoroughly washed and then rinsed with distilled water to avoid any biological or physical contamination from the preceding experiment. Furthermore, all sand and gravel used in any previous experiments as supporting material was replaced with clean sand and gravel. The final step before the sludge was poured into the columns was to saturate the sand with distilled water. This prevented impedance of flow due to air pockets.

The first three experiments were designed to test a settled sludge condition. Basically, the sludges were introduced into the columns with an additional depth of supernatant applied, thus providing a greater potential filtrate volume, and, in essence, constituting a facsimile of a lagoon.

Experiments IV-VI used a homogeneous charge of sludge without an applied supernatant. In Experiment IV, all nine columns had an initial head of 54 cm with Franklin, Ottawa, and Hermitage sands used as the supporting media in columns 1-3, 4-6, and 7-9 respectively. In Experiment V, Franklin sand was again the supporting media for columns 1, 2, and 3; Ottawa sand for columns 4, 5, and 6; and Hermitage sand for columns 7, 8, and 9. However, the initial head varied--44 cm for columns 1, 4, and 7; 74 cm for columns 2, 5, and 8; and 114 cm for columns 3, 6, and 9.

Once all nine columns were filled to their predetermined head, the stoppers were released and the drainage begun. During the course of

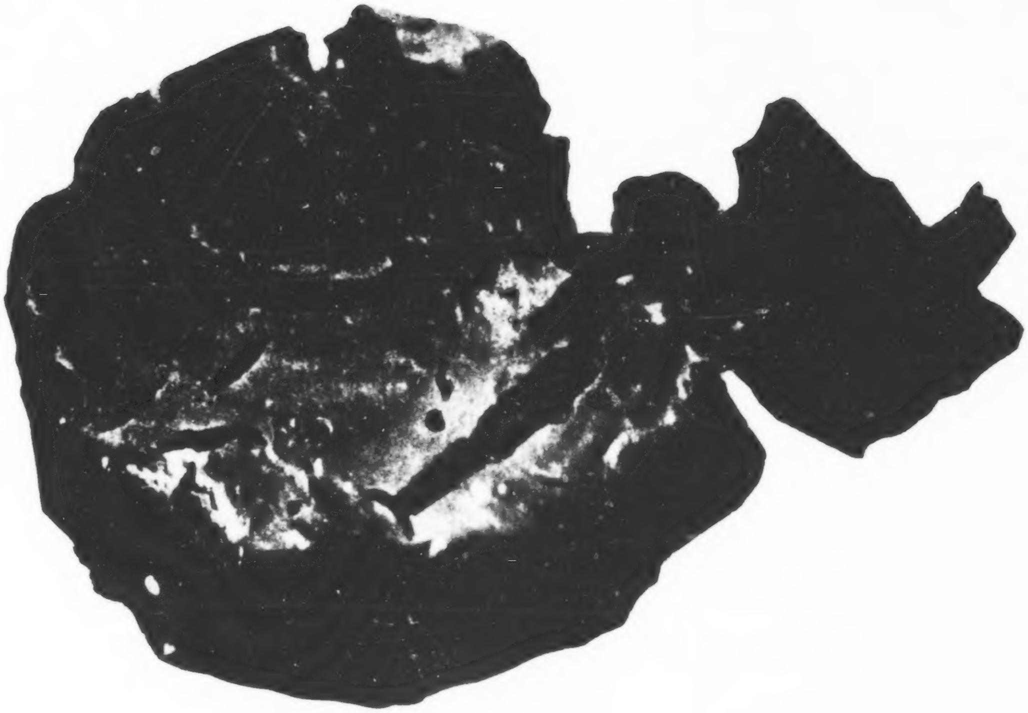


Figure 17: A sludge cake formed resulting from gravity dewatering only. From Experiment V.

the six experiments, the volume of filtrate drained from the sludge in each column was measured and recorded three times daily. The length of each experiment varied from 20 to 57 days depending on the time required for the drainage of natant to cease in the majority of the columns leaving a dewatered sludge cake (Figure 17).

For Experiment VI, only one column was used (Figure 15). Nine piezometer tubes were placed along the length of the column so that the pressure at different depths of sludge could be studied. To prevent the sludge clogging the piezometer tubes, the column was filled with distilled water likewise filling the piezometer tubes. The tubes were stoppered, forcing the distilled water to remain in the tubes after the column was emptied. The sludge was then poured into the column after which the stoppers on the piezometer tubes were released. The initial head was 96 cm.

The reader is referred to Tables 1 and 2 for further information on experiment variables.

Experiment Number	Source of Sludge	Treatment Process	Solids Content %	Grease Content % Dry Solids	pH	Specific Resistance at h_c=34.4 cm. $sec^2/gm.$	Coefficient of Compressibility
I	Old Hickory, Tennessee	Digested primary and secondary sludge following trickling filter	6.35	6.76	7.4	$1.95X10^{10}$	0.51
II	Old Hickory, Tennessee	Digested primary and secondary sludge following trickling filter	5.00	3.78	6.6	$1.45X10^9$	0.82
III	Madison, Tennessee	Digested primary and secondary sludge following trickling filter	4.89	3.83	6.8	$1.9X10^9$	0.74
IV	Franklin, Tennessee	Digested primary and secondary sludge following activated sludge process	3.70	2.80	--	$8.25X10^9$	0.66
V	Franklin, Tennessee	Digested primary and secondary sludge following activated sludge process	2.78	--	6.9	$3.5X10^9$	0.64
VI	Franklin, Tennessee	Digested primary and secondary sludge following activated sludge process	3.50	--	--	$5.5X10^9$	0.71

Table 1: Characteristics of Sludges Used

Experiment Number	Column Number	Type of Sand	Depth of Sand (cm)	Depth of Gravel (cm)	Total Negative Head (cm)	Initial Depth of Sludge Solids (cm)	Initial Total Head (cm)	Duration of Experiment (days)	Final Depth of Sludge Solids* (cm)	Final Total Head** (cm)
I	1	H	20	4	51.9	5	63.5	20	1	52.2
	2	O	20	4	51.9	5	63.5	17	1	52.5
	3	F	20	4	51.9	5	63.5	20	1	52.3
	4	H	20	4	51.9	5	80.5	20	-	52.0
	5	O	20	4	51.9	5	80.5	11	-	54.1
	6	F	20	4	51.9	5	80.5	20	-	65.0
	7	H	20	4	51.9	5	99.5	20	-	58.5
	8	O	20	4	51.9	5	99.5	20	-	73.2
	9	F	20	4	51.9	5	99.5	20	-	82.5
II	1	H	4	2.5	33.0	12	50.0	14	2.8	34.5
	2	O	4	2.5	33.0	12	50.0	14	2.7	35.5
	3	F	4	2.5	33.0	12	50.0	14	2.9	35.1
	4	H	4	2.5	33.0	12	70.0	16	5.8	38.0
	5	O	4	2.5	33.0	12	70.0	14	2.7	34.6
	6	F	4	2.5	33.0	12	70.0	16	2.8	34.7
	7	H	4	2.5	33.0	12	90.0	16	2.5	42.4
	8	O	4	2.5	33.0	12	90.0	16	2.8	39.1
	9	F	4	2.5	33.0	12	90.0	16	3.2	39.3
III	1	F	14	1.8	35.0	7	69.0	8	-	38.5
	2	F	14	1.8	35.0	7	69.0	8	-	37.8
	3	F	14	1.8	35.0	7	69.0	8	-	44.4
	4	O	14	1.8	35.0	7	69.0	8	1	38.2
	5	O	14	1.8	35.0	7	69.0	8	1	37.0
	6	O	14	1.8	35.0	7	69.0	8	1	37.7
	7	H	14	1.8	35.0	7	69.0	8	1	37.6
	8	H	14	1.8	35.0	7	69.0	8	1	37.6
	9	H	14	1.8	35.0	7	69.0	8	1	37.0

*Estimated thickness after some consolidation. **Determined from filtrate collected.

Table 2: Classification of Experiments

Experiment Number	Column Number	Type of Sand	Depth of Sand (cm)	Depth of Gravel (cm)	Total Negative Head (cm)	Initial Depth of Sludge Solids (cm)	Initial Total Head (cm)	Duration of Experiment (days)	Final Depth of Sludge Solids* (cm)	Final Total Head** (cm)
IV	1	F	4	2	34 0	20	54.0	27	7.0	41.1
	2	F	4	2	34.0	20	54.0	27	7.0	41.3
	3	F	4	2	34.0	20	54.0	27	6.0	40.0
	4	O	4	2	34.0	20	54.0	27	6.0	40.8
	5	O	4	2	34.0	20	54.0	27	7.0	41.1
	6	O	4	2	34.0	20	54.0	27	7.0	41.3
	7	H	4	2	34.0	20	54.0	27	7.0	40.7
	8	H	4	2	34.0	20	54.0	27	6.0	40.4
	9	H	4	2	34.0	20	54.0	27	6.0	40.0
V	1	F	3.5	3	33.0	11	44.0	8	3.2	35.1
	2	O	3.5	3	33.0	11	44.0	8	2.3	35.4
	3	H	3.5	3	33.0	11	44.0	9	1.0	34.1
	4	F	3.5	3	33.0	41	74.0	34	19.1	56.0
	5	O	3.5	3	33.0	41	74.0	57	18.5	51.7
	6	H	3.5	3	33.0	41	74.0	57	18.9	51.9
	7	F	3.5	3	33.0	81	114.0	57	-	85.3
	8	O	3.5	3	33.0	81	114.0	57	-	83.1
	9	H	3.5	3	33.0	81	114.0	55	-	72.9
VI	1	O	3.5	3	34.0	62	96.0	15	41.0	86.9

*Estimated thickness after some consolidation. **Determined from filtrate collected.

Table 2 (Continued): Classification of Experiments

CHAPTER VIII

DISCUSSION OF RESULTS

The theoretical formula (Eq. 25) previously derived has shown that the drainage rate of a wastewater sludge on sand or other highly permeable material is dependent on the solids content of the sludge, the head, specific resistance, coefficient of compressibility, filtrate density, and dynamic viscosity of the filtrate. The degree of accuracy with which this formula fits the experimental data will in part be dependent upon the accuracy with which the dependent variables are measured.

Experimental errors incurred in the determination of the solids content must be considered minimal (<5%) as the procedure utilized multiple samples (see appendix). The solids determinations were used in both Equation 25 and in the determination of specific resistance, and here values inserted were in fact approximations for the volume of filtrate per unit weight of solids. Greater accuracy in laboratory determinations for solids content was thus not justified.

The initial head was directly measured to 1 mm. Instantaneous values for head were calculated from the initial head and the instantaneous volume of filtrate. The volume could be measured to within 2 ml which represented a head interval of 2.5 mm, certainly not a source of significant error. A greater error occurred at times with the appearance of bubbles in the filtrate discharge tubes below the nozzles. Sometimes the bubbles attained 2-3 cm length, thus reducing correspondingly the calculated head. However, the heads were invariably greater than 30 cm, and hence the bubbles created minimum disturbance.

Values for filtrate density and dynamic viscosity were originally checked and adjudged to be identical to values for water at the same temperature. A series of preliminary tests indicated this conclusion was sound, as any deviation of filtrate characteristics from water were clearly smaller than the effects of temperature fluctuation. These fluctuations, minimized by the air-conditioning system in the laboratory, approximated ±3°C during the course of an experiment. Mean temperatures were 24°C, leading to values of 0.00919 gm/cm-sec for dynamic viscosity, and 1.0 gm/cm^3 for density, which appear in both the calculations for gravity dewatering time and specific resistance.

Specific resistance is calculated from the slope of t/V versus V as well as the solids content, filtrate viscosity and density, pressure, and area of the funnel. Only the slope, area and pressure need be discussed here, the other factors have been previously just mentioned.

The accuracy of slope determination was generally quite good (see Figure 11 and appendix). At low values of V, a linear relationship was not seen. This is a common occurrence as substantiated in the literature, and is due to the absence of a filter cake at t=0; at which point the specific resistance theory does not apply. Note that t/V at t=0 equals 0/0, or an indeterminate. Once a cake is established, the theory applies, and a straight line develops. The graphs demonstrate this. Deviations from a line within the linear zone were due to difficulties in filtrate collection. The filtrate did not pour evenly into the burette, but spasmodically, in keeping with the spasmodic dripping from the back-face of the Buchner funnel perforated filter. Pressure fluctuations during each test were not greater than 1 cm during the time periods of significance, and were momentary. The pressure readings were to 5 mm.

Coefficient of compressibility determinations as shown in Figure 14 and the appendix corroborate the empirical relationship between specific resistance and pressure. It appears that the values for the coefficient as determined could have errors of 10% or greater. Note that the values for R_c at $\Delta p = \Delta p_c$ is taken from the respective lines of best fit, in a sense averaging the values of specific resistance determined for each experiment. The values for specific resistance at the commonly reported pressure of 15 in. Hg (=517 cm H_2O), and the coefficients of compressibility from the six experiments are shown in Table 3. They compare favorably with values from other researchers.

In Experiments I - III, the theoretical equation developed (see appendix) to describe a lagoon dewatering could not be proven due to a paucity of reliable data. In most instances a phenomenon of "surging" occurred, whereby an extremely high rate of dewatering voided the natant during the early days of each experiment. This will be described in greater detail elsewhere. Also, great difficulty

Sludge	Specific Resistance (sec^2/gm)	Vacuum ($dynes/cm^2$)	Coefficient of Compressibility	Corrected S.R. for 15 in. of Hg (sec^2/gm)	Reference
Digested*	-	-	.74	$1.42X10^{10}$	Coackley (34)
Digested*	-	-	.64	$1.26X10^{10}$	Coackley (34)
Digested mixed trickling filter	$2.04X10^{10}$	$4.0X10^5$	.63	$2.38X10^{10}$	Coackley (4)
Digested mixed trickling filter	$1.63X10^{10}$	$4.0X10^5$	.56	$1.87X10^{10}$	Coackley (4)
Raw	$0.398X10^{10}$	$4.9X10^5$	.76	$0.407X10^{10}$	von Niemitz (18)
Digested primary	$2.17X10^{10}$	$4.9X10^5$	-	-	Water Poll. Lab. (33)
Raw	$1.9X10^{10}$	$4.9X10^5$	-	-	Water Poll. Lab. (33)
Digested mixed activated	$2.9X10^{10}$	$4.9X10^5$	-	-	Water Poll. Lab. (33)
Raw	-	-	.54	$4.7X10^9$	Eckenfelder and O'Connor (35)
Digested mixed trickling filter	-	-	.51	$7.24X10^{10}$	Experiment I
Digested mixed trickling filter	-	-	.82	$1.20X10^{10}$	Experiment II
Digested mixed trickling filter	-	-	.74	$1.40X10^{10}$	Experiment III
Digested mixed activated	-	-	.66	$4.8X10^{10}$	Experiment IV
Digested mixed activated	-	-	.64	$2.1X10^{10}$	Experiment V
Digested mixed activated	-	-	.71	$3.8X10^{10}$	Experiment VI

*Not identified other than digested.

Table 3: Experimental Data

was encountered with the placing of supernatant on top of the thick sludge. Turbulence caused a mixing of the two, with a poor, partial separation developing after 24 hours at which time each experiment began. The initial solids content, specific resistance, and coefficient of compressibility were determined from the sludge initially applied, and as shown, these values were to be used in the theoretical equation. Unfortunately they were clearly altered by the addition of supernatant. It might also be mentioned that additional settling after drainage began did not improve the data. Additional settling was minor due to the short drainage times, and also due to temperature currents. Experiments I - III were run in late summer with the columns some 2 feet from an external wall. The outdoor heat created a warmer backside on the columns leading to a flow pattern inimical to settling. Later experiments run in the fall did not experience this problem.

Quon (20) witnessed surges which he believed were caused by the absorption of the air in the sand by the filtrate, thus increasing the effective porosity, which in turn increased the rate of drainage. Since the supporting media in every experiment was saturated with water before each experiment, Quon's observation does not appear to apply here.

Visual observations of the surging showed that the filtrate turbidity sharply increased as a result. Further, it was noted that larger sludge particles penetrated into the sand at the edges of the column-sand interface. This leads one to suspect that the sand-column interface served as an easier exit for particles and that the column diameter may influence the occurrence if not magnitude of the surges. The relatively thin layers of sludge in Experiments I - III exhibited a "punching" of the surface near the edges, adding credence to this theory.

Further substantiation was indicated by Experiments IV - VI, which were charged with homogenous mixtures providing thicker sludge cakes. Surging was significantly reduced, and it may be assumed that this was due to lower pressure gradients across the sludge cakes. It should also be mentioned that the disturbance of the sand surface during the filling with sludge alone, was definitely less than in those cases where supernatant was then later added. An uneven sand surface would probably encourage surging, for a cake would build of uneven thickness.

In Experiment VI, only one column was used. This column had 10 piezometer tubes running the length of the columns to be used as a measure of the pressure differences in the sludge column. Initial head of the sludge on the Ottawa sand was 96 cm. During the experiment it was proven that a large pressure difference which is proprotional to the head loss occurred at the sludge-sand interface. Therefore, the build-up of a thin sludge cake was responsible for the majority of the head loss in the column and would thus be the determining factor which controlled drainage rates. This follows standard filter theory for compressibile materials, hence justifying the assumption made during the theoretical derivation that the drainage rate was proportional to the build-up of a thin sludge cake. This is contrary to the previous assumption that the drainage flow is resisted throughout the depth of sludge (11), and not just the few millimeters of sludge at the sludge-sand interface.

After the completion of Experiment VI which provided proof of the existence of a thin sludge layer at the sludge-sand interface, a theory was proposed to explain the surging phenomenon. This layer through structural rigidity and adsorption bridged the distances between the individual sand particles. If the distance between particles were large, a small amount of stress (from viscous drag) would cause a break in the sludge cake to occur, allowing an early release of filtrate. As the liquor passed through, new sludge particles would eventually fill the break and stop the surge. However, this may await a low drainage rate at which viscous drag was a minimum, i.e., possibly the passages were plugged only at the end of dewatering. If the distances between the individual sand particles were small, more stress would be needed to collapse the bridging sludge cake and once a break occurred less solids would be needed to fill the break.

To prove the rigid sludge cake theory, the effective size of each sand was measured since the effective size was an approximate measure of the sand grain diameters. A small effective size would indicate a small distance between individual sand particles and a large effective size would indicate a large distance between particles. With reference to Table 4, it was found that Franklin sand possessed the smallest effective size and that the Hermitage sand, the largest. The sludge supported by Franklin sand in Experiment IV did not surge in the twenty-five days (Figure 18) of the experiment except for

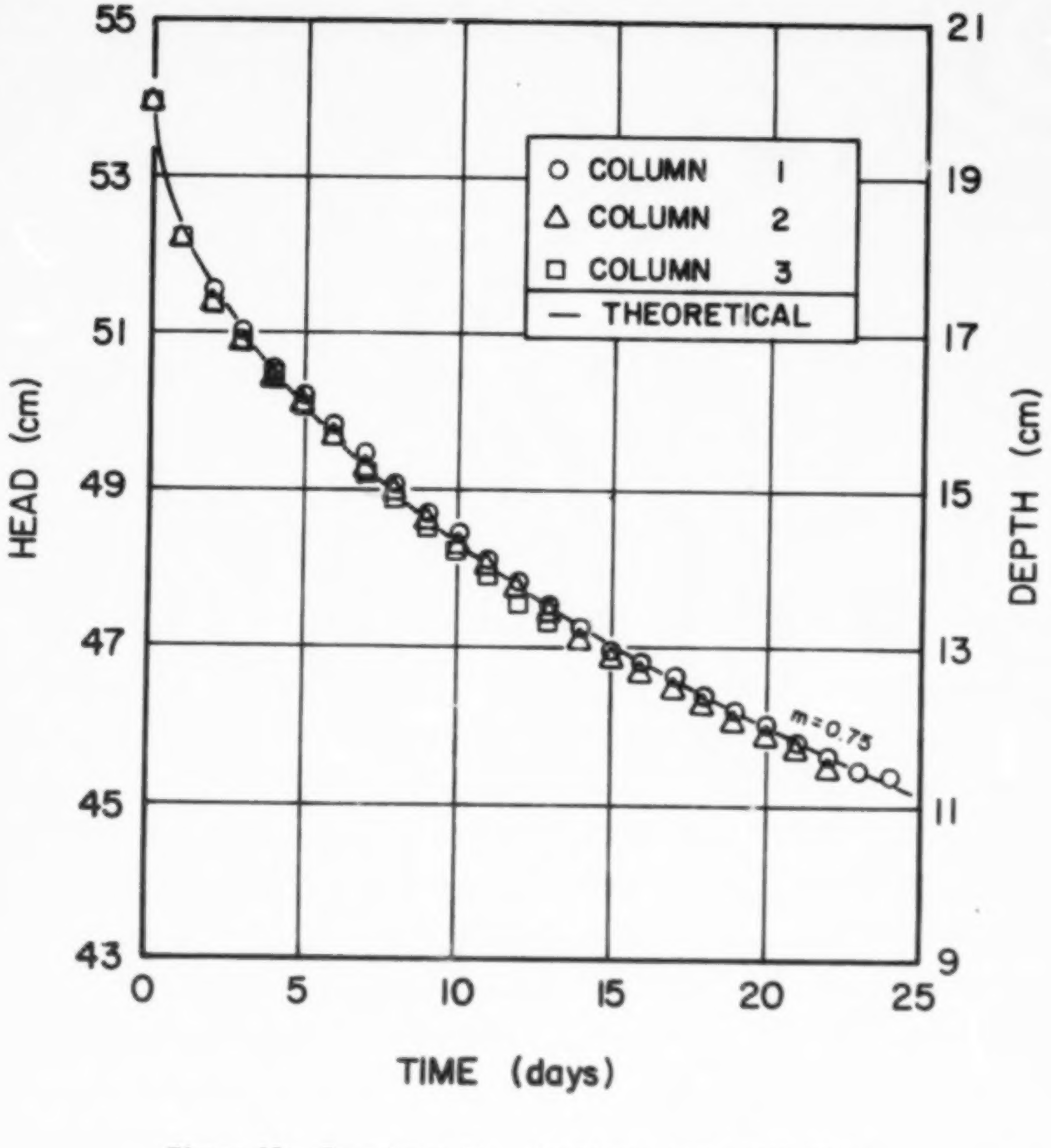


Figure 18: Experiment IV. Triplicate tests with 20 cm of sludge on Franklin sand.

column 3 on the 14th day. The sludge supported by the Ottawa sand (Figure 19) surged in all three columns around the 17th day of the experiments. And finally, the sludge supported by the Hermitage sand surged in the vicinity of the 14th day (Figure 20).

Data from Experiments IV and V, plotted in Figures 18 - 23, presented sufficient data before surging to test the validity of Equation 25. The theory, however, predicted longer drainage times in all cases, but a study revealed that a constant correction factor applied for each type of sand.

One would not expect the value of R_c, calculated from Buchner funnel testing, to fully represent a value of specific resistance of a sludge on different supporting media such as a sand. Approximately 40-50% of the gross sand surface is porous; that of a Buchner funnel less so, even though filter paper is used. To illustrate extreme cases of the role of the supporting media, it is clear that impermeable supporting media will allow no dewatering, whereas very coarse supporting media will retain no solids, resulting in a zero resistance to flow.

To account for the interrelationship between the sludge and the supporting media, but mindful of dimensional correctness, a media factor must be introduced into Equation 25. This media factor can be considered as a function of the ratio of a representative sludge floc diameter to an equivalent diameter of a sand grain: the sand grain representing the effect of the supporting media or some equivalent parameter of the Buchner funnel, or filter leaf, or any other dewatering device. The value of the media factor, m, would be larger for finer

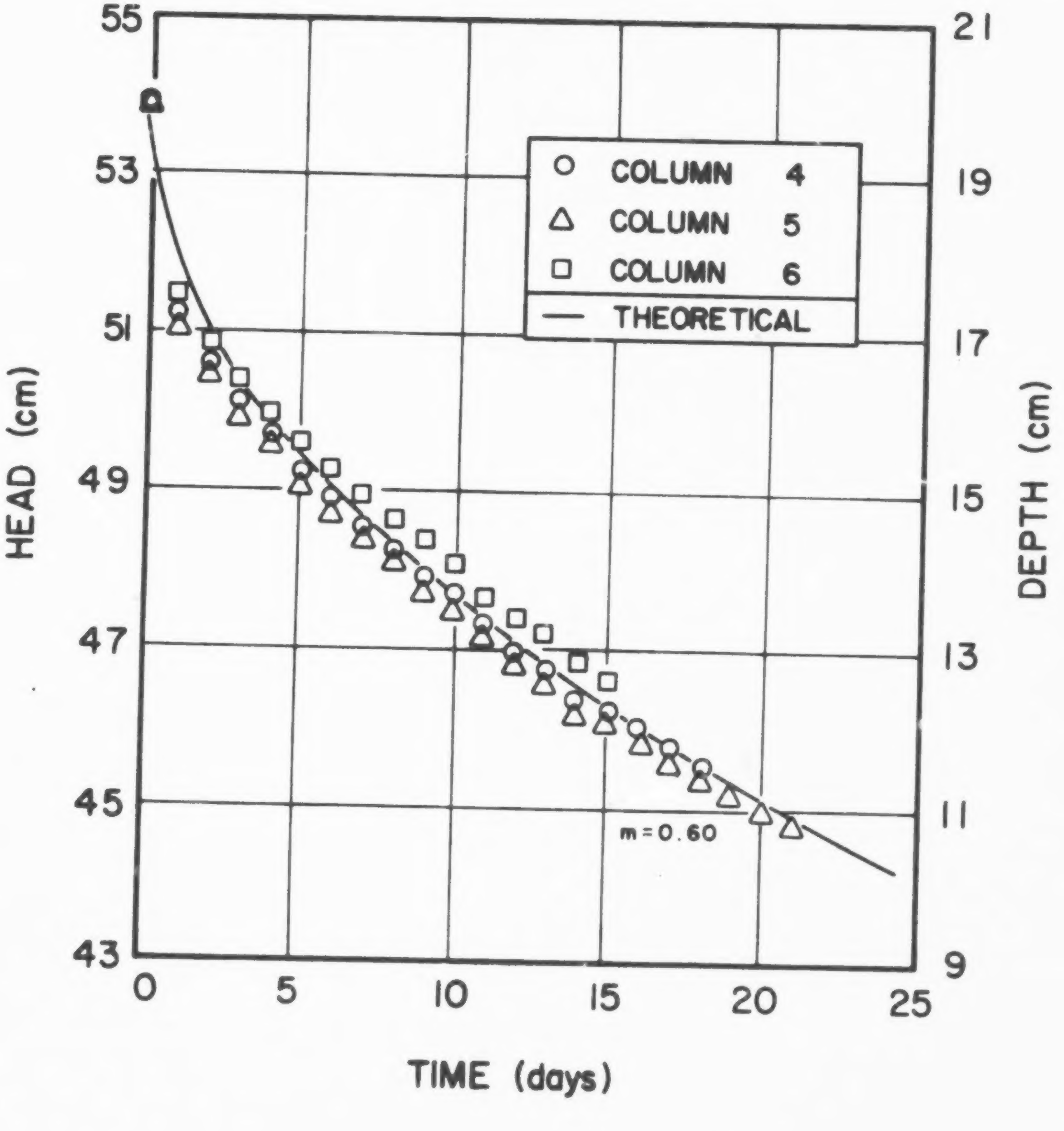


Figure 19: Experiment IV. Triplicate tests with 20 cm of sludge on Ottawa sand.

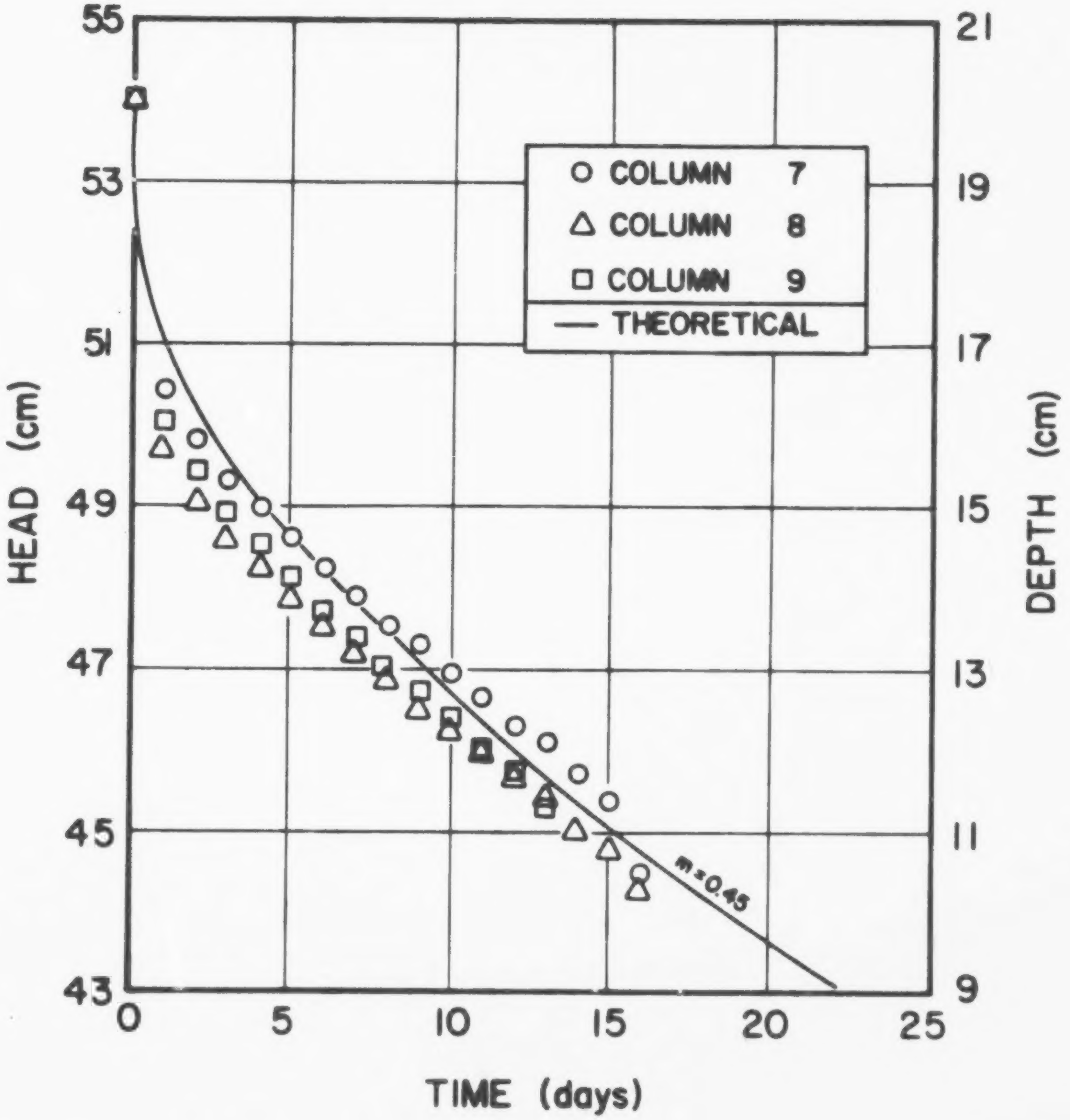


Figure 20: Experiment IV. Triplicate tests with 20 cm of sludge on Hermitage sand.

sands and smaller for coarse sands. As a function of ratio of diameters, m would be dimensionless.

The values found for the media factor by curve fitting, and listed in Table 4, exhibit the correct directional relationship with the corresponding D_{10} size; that is, the decrease in media factor (or drainage time) results from an increase in the sand size. Such an increase in sand size, however, inevitably increases the turbidity and color of the filtrate (see Figure 25).

Sand Source	Sand Designation	D_{10}(mm)	D_{60}/D_{10}	Media Factor
Franklin Treatment Plant	F	0.16	1.25	0.75
Ottawa Standard Sand	O	0.60	1.23	0.60
Hermitage Hill Treatment Plant	H	0.78	1.41	0.45

Table 4: Physical Characteristics of Supporting Sand

A plot of the experimental data and theoretical data (Figure 21, Figure 22, and Figure 23) substantiated the conclusions drawn from Experiment IV that the mathematical model (Eq. 25) with the media factor could approximate the actual drainage rate of a sludge on a sand bed and that the type of sand did affect the initial filtrate volume and the time of surge.

The results obtained from Experiments IV and V proved the applicability of (Eq. 25) to predict the drainage rates of a sludge on a sand bed. The sludge supported by the Franklin sand in Experiment IV drained according to the theoretical curve (Figure 18). The sludge

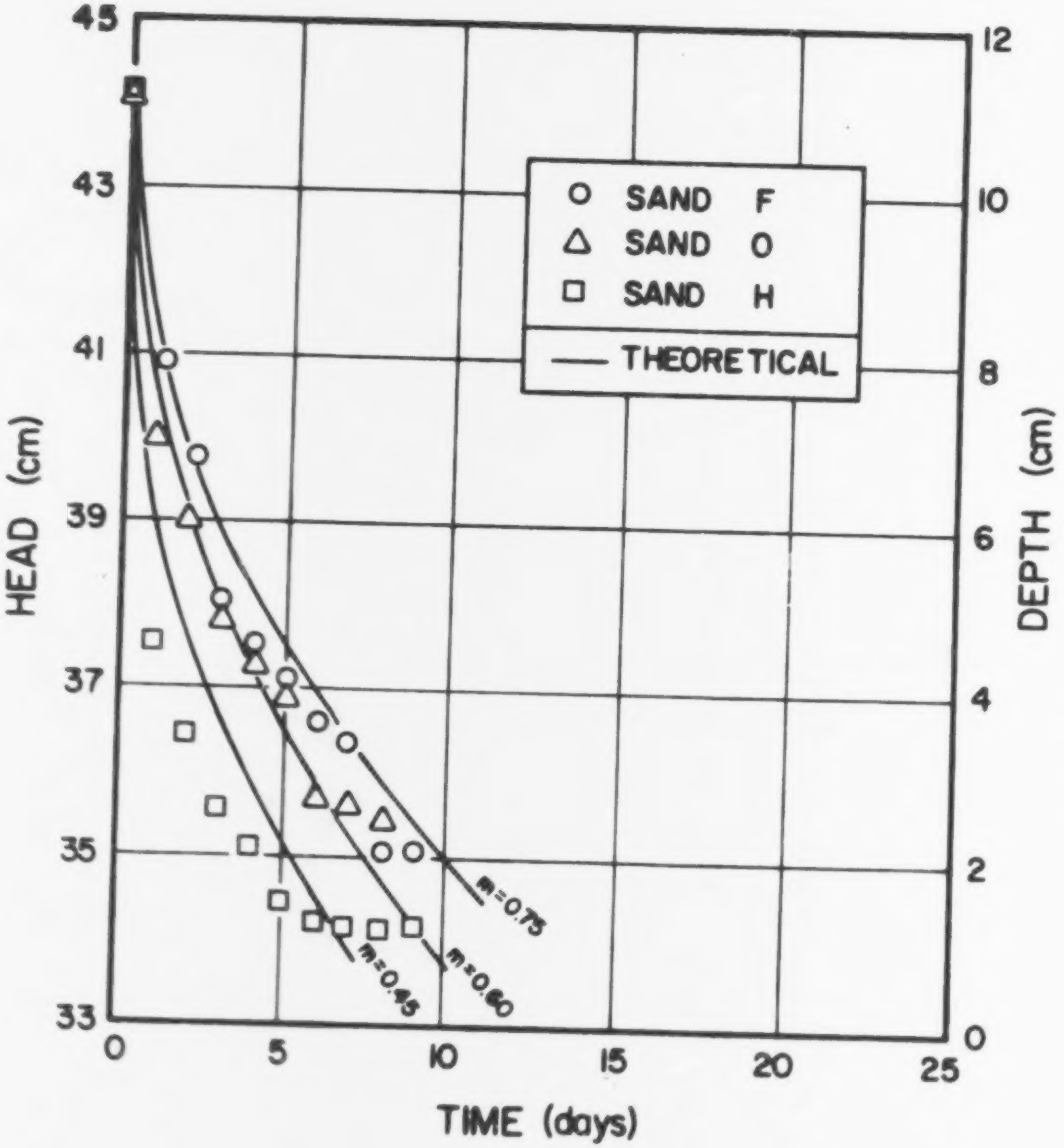


Figure 21: Experiment V. 11 cm of sludge applied on three different sands.

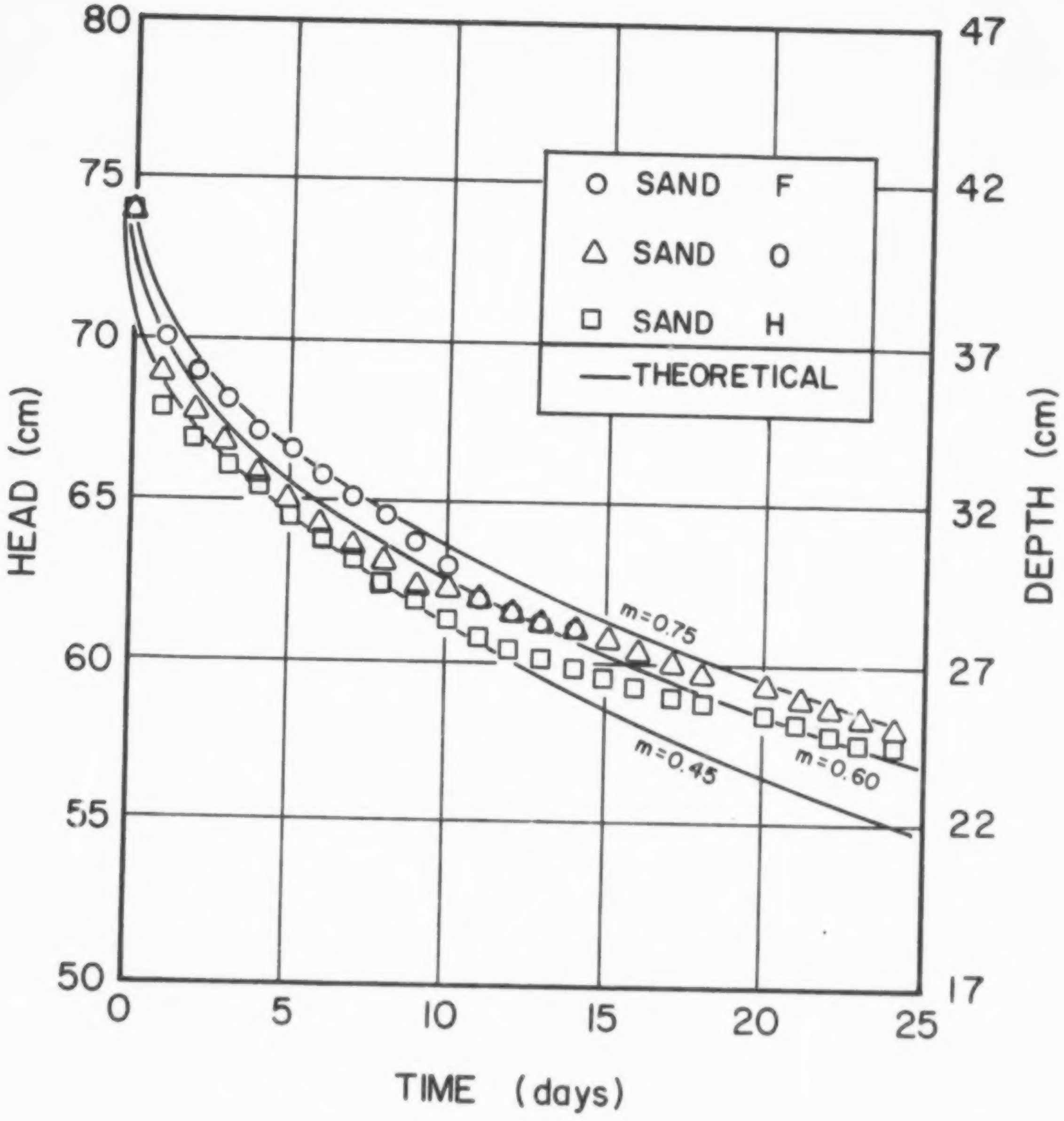


Figure 22: Experiment V. 41 cm of sludge applied on three different sands.

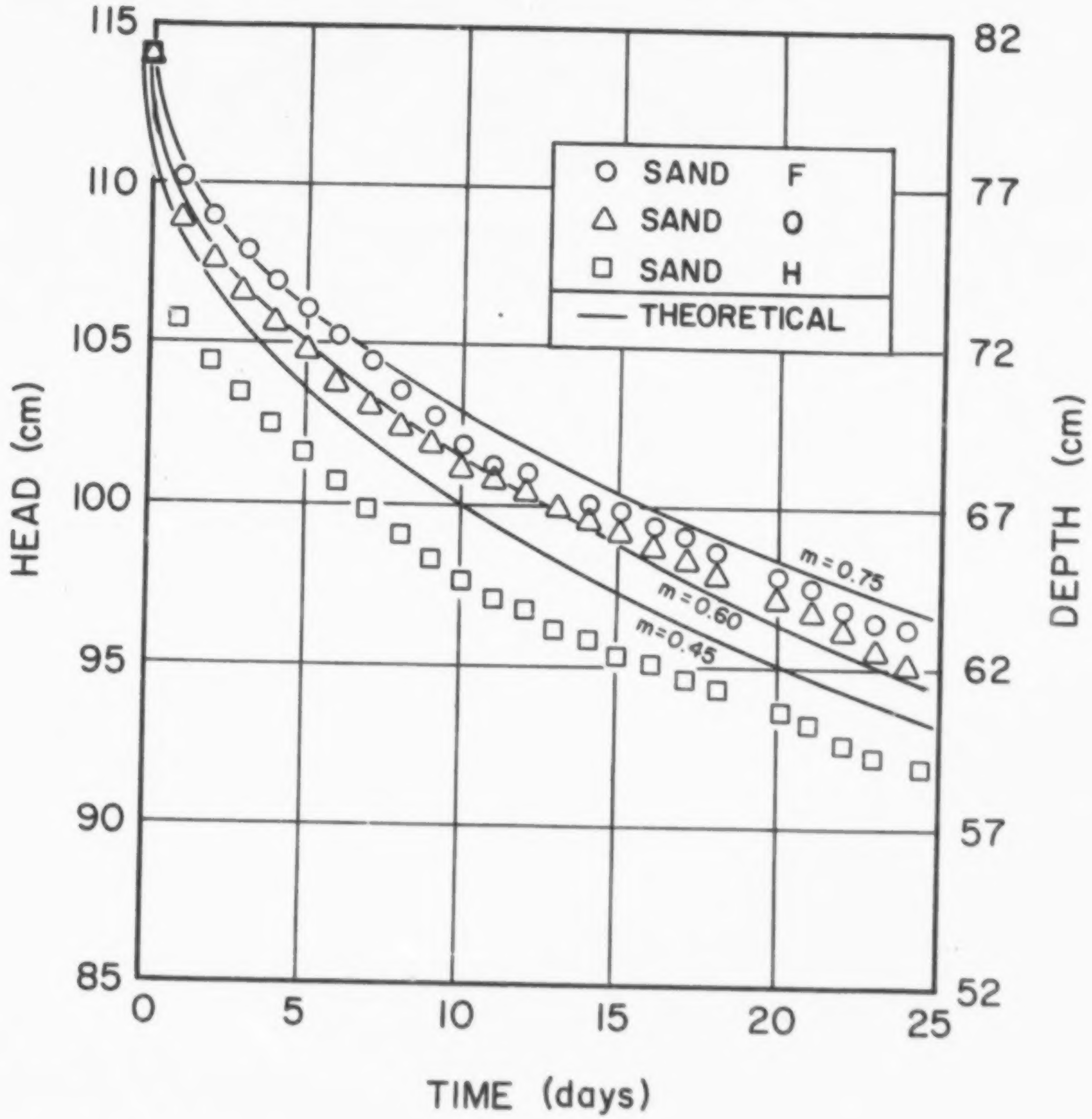


Figure 23: Experiment V. 81 cm of sludge applied on three different sands.

supported by the Ottawa sand because of an initial filtrate release drained at the rate determined by (Eq. 25) but plotted below the theoretical curve in the first few days (Figure 19). If the volume of the initial filtrate released divided by column area were added to each point plotted in Figure 19, the experimental curves would coincide with the theoretical curve. The sludge supported by the Hermitage sand also had an initial unpredicted filtrate release which offset the experimental curve from the theoretical curve.

The initial high drainage rates resulted from an initial retarded cake formation due to some of the solids being drawn into the supporting media instead of being retained above the sand interface. During the course of the six experiments, there has been strong evidence that the initial filtrate released as well as the time of surging was related to the supporting media. The sludge supported by the Franklin sand not only had very little filtrate released but also did not surge in 25 days. However, the same sludge supported by the Hermitage sand did release a sizeable amount of filtrate at the beginning of the experiment as well as surging in approximately 14 days.

As previously mentioned, if the head loss due to the initial filtrate release were added to each point on the head versus time curves for the Ottawa and Hermitage sands respectively, the curves would then coincide with the head versus time curve for the Franklin sand with the same initial head, thus justifying identical media factors for all the sands tested. A statistical test in the appendix demonstrates this procedure (see Figure 24).

Figure 21 should also be noted for the complete drainage at 6 days

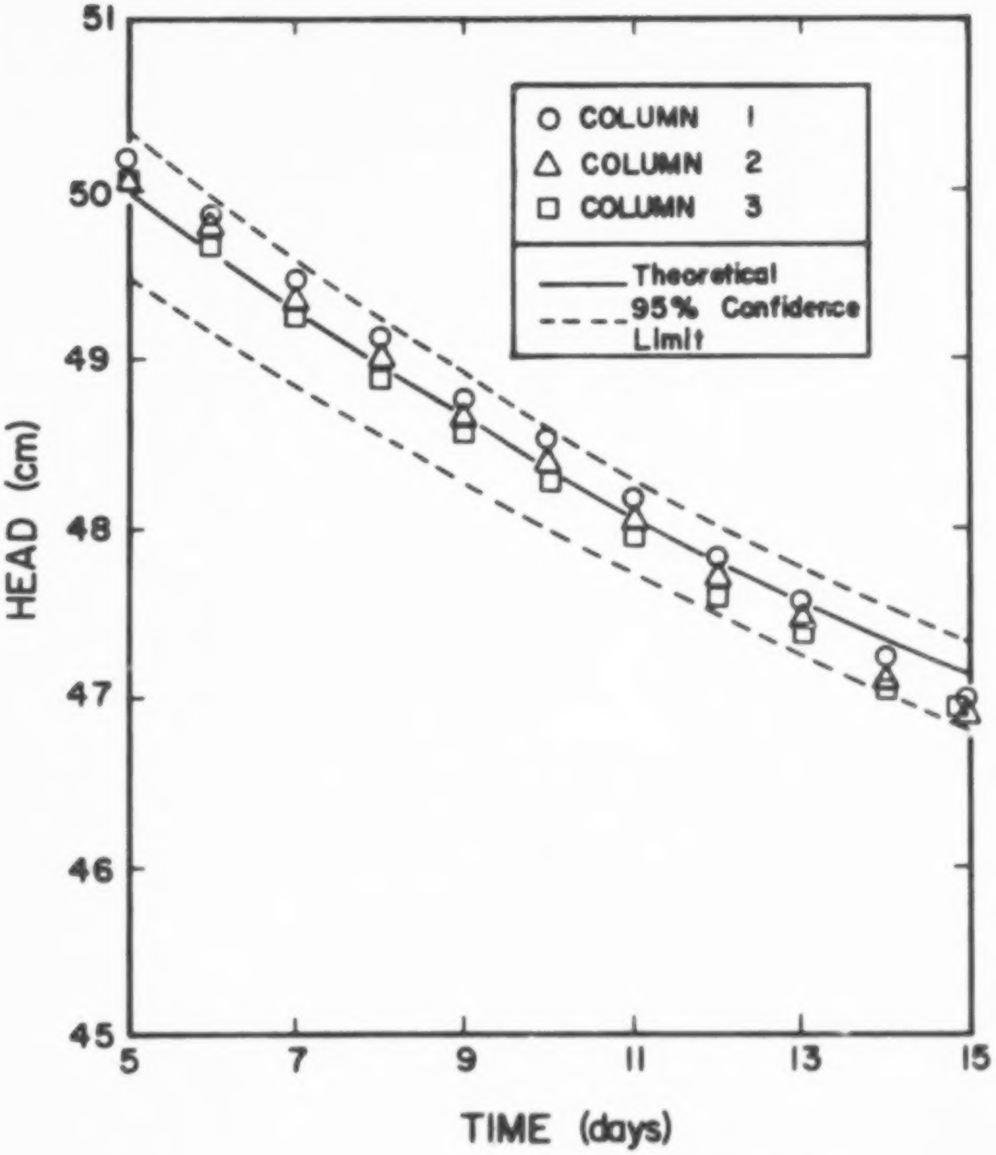


Figure 24: A plot of the 95% confidence interval with data from column 1, 2, and 3 in Experiment IV.

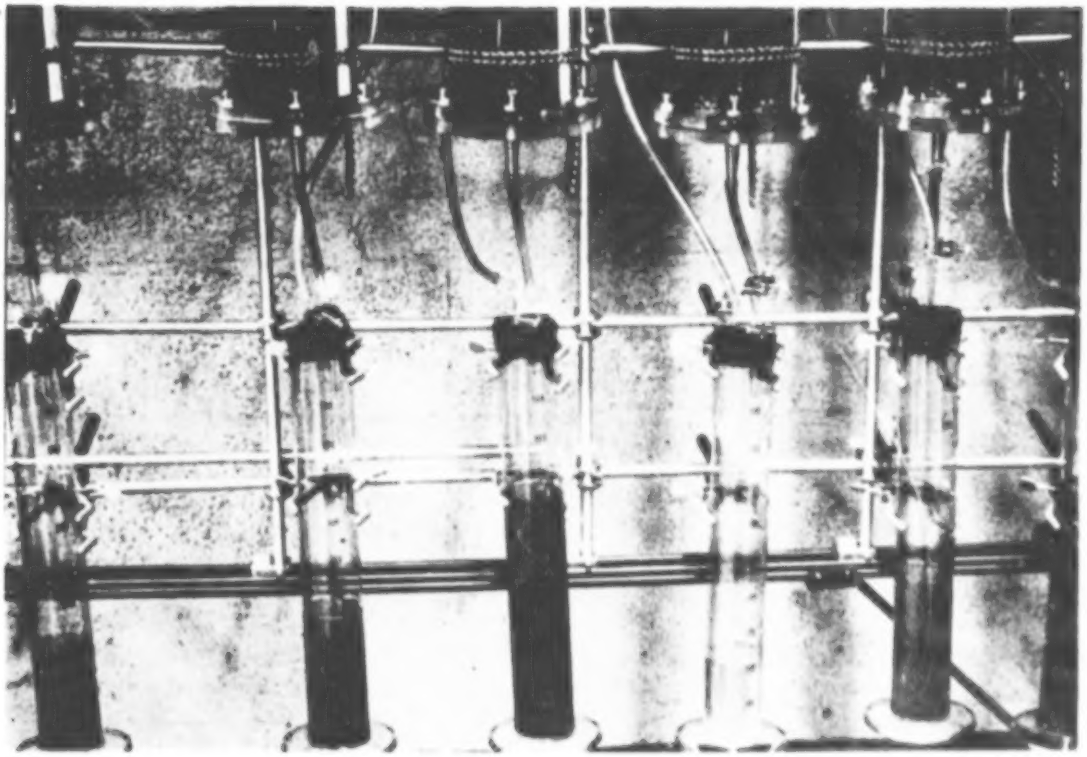


Figure 25: Filtrate collection during the first 3 days showing relationship of filtrate to supporting media (sand).

From left to right, the first and fourth graduated cylinders exhibit a small but clear volume of filtrate; the third and sixth, a larger volume of filtrate but one of much higher turbidity. The second and fifth cylinders show intermediate results. Respectively, the sands used are types F, G, and H, representing effective sand sizes of 0.16, 0.60, and 0.78 mm.

that occurs in the Sand H run. At this point the sludge cake has about 30% solids, the liquid interface has reached the sludge cake surface, and the menisci in the sludge cake resist the suction of the underlying water. The dewatering theory does not describe the final consolidation process of the sludge cake.

CHAPTER IX

SIGNIFICANCE OF RESULTS AND CONCLUSIONS

The theoretical formula (Eq. 25) developed herein for describing gravity dewatering of wastewater sludge was found to be experimentally verified. Initial solids content, head, specific resistance, coefficient of compressibility, and filtrate viscosity and density were found to be parameters affecting the time required for dewatering. The experiments also prove the existance of a dimensionless media factor, m, which related the sludge dewaterability on sands and other filter media to its dewaterability in the Buchner funnel.

Further research on the influence of the media on drainage is clearly indicated. Efforts should be made to correlate some sand characteristic such as effective size to the media factor. Further investigation should also be made to determine whether the media factor is a multiplier as used in this research, or if it applies better as an additive factor to the model. In the latter case, a corrective multiplier would still be used to correlate the vacuum test with the gravity drainage. For a Buchner funnel test as described here, the multiplier was seen to be about 0.75. Probably a different factor would apply for a filter leaf test which also is commonly used to measure specific resistance.

The occurrence of surging at unpredictable times is of concern, although it places the mathematical model on the side of conservatism. It is felt, however, that full-scale drying beds do not develop surges, as information regarding surges in the literature appears limited to pilot-plant tests. Possibly the diameter of the column influences the occurrence of surges in the test results. This should be clearly substantiated.

It should be noted that the gravity dewatering parameter of head is related to the depth of sludge applied. Quite possibly they are synonymous in actual drying bed operation: the presence or absence of a negative head, or suction, beneath the sludge-sand interface should be investigated.

Without a negative head, or with an assumed or determined value, the mathematical model can be adapted to directly calculate bed loadings. It could then be used to calculate the area needed to dewater a given amount of sludge by gravity drainage alone. The area calculated would have an adequate safety factor to prevent underdesign since other mechanisms such as evaporation, flotation, transpiration, and surging could increase moisture removal rates and thus decrease area required. The mathematical model in conjunction with a mathematical model describing evaporation (17) could provide the environmental engineer with a badly needed rational method to determine sand bed areas. Perhaps more importantly, it could provide a rational basis for the design engineer to compare drying bed performance to other means of dewatering. The justification, or lack thereof, for enclosures or roofing over drying beds could also be reanalyzed.

As a further development, the mathematical model can be used to determine the most effective depth of application. A depth could be calculated which would allow a sludge to dewater to "forkable" condition in a specific amount of time. The mathematical model developed in this research may be used to measure the effectiveness of the many chemical coagulants including the polyelectrolytes added to wastewater sludge to increase dewatering rates. Not only may the different coagulants be quantitavely compared by means of the model, but also the model may be used to determine the most suitable coagulant dosage for given conditions. For example, if a sludge to be applied on a sand bed would dewater (using the model to determine the time) in 30 days, but there are only 22 days between sludge applications, the operator could calculate the required dosage of coagulant to reduce dewatering time from 30 days to 22 days.

Unfortunately, there are not many wastewater treatment plants at the present time that could spare the time or money to measure the most effective coagulant dose for the sludges to be applied on the sand drainage beds. However, in the future, with the centralization and expansion of wastewater treatment plants to fit the needs of an ever increasing population, the mathematical model could conceivably be used to determine coagulant dosages, since the money saved could far exceed the cost necessary to determine the optimum coagulant dosages. If a nomograph based on the mathematical model could be developed, plant operators with a minimum of experimental work would be able to make more effective use of the sand bed area at their disposal.

Although there exists a great deal of sludge conditioning data in the literature, the data are almost invariably expressed solely in terms of specific resistance, omitting any reference to the coefficient of compressibility. It therefore appears that new research is particularly needed to determine the effect of conditioners, not only on specific resistance, but on the coefficient of compressibility as well. The net effect of conditioners must then be evaluated through use of the mathematical model developed herein.

APPENDIX I

DERIVATIONS

A. DERIVATION OF THE MATHEMATICAL MODEL FOR A SETTLED SLUDGE

It has been proven in the previous derivation that a sludge which is not presettled will dewater at a rate described by the differential equation

$$\frac{dh}{dt} = - \frac{1}{\mu} \frac{h}{F} \frac{100}{R_c (H_o - h)} \left(\frac{h_c}{h}\right)^{\sigma}$$

For a special case when there is a presettled sludge with $h_t = H_o$ - (thickness of the presettled sludge) the following integration can be made when $h_t < h$.

$$\int_o^{t_t} dt = - \frac{\mu R_c F}{100 (h_c)^{\sigma}} \int_{H_o}^{h_t} (H_o - h) h^{\sigma - 1} dh$$

$$t_t = \frac{\mu R_c F}{100(\sigma+1)\ (h_c)^{\sigma}} \left[h_t^{\sigma+1} - \frac{\sigma+1}{\sigma} H_o\ h_t^{\sigma} - H_o^{\sigma+1} + \frac{\sigma+1}{\sigma} H_o^{\sigma+1} \right] \quad \text{(Eq. A 1)}$$

t_t is equal to the time required for the initial head to drop a distance equal to the depth of the presettled sludge. Once the head has decreased to h_t, according to theory, there will not be any additional sludge solids accumulating at the sludge-sand interface. However, compression of the existing filter cake will continue, and thus affect the drainage rates.

When $h < h_t$

$$V = (H_o - h_t)A = \text{constant } (L^3)$$

Therefore

$$\frac{dh}{dt} = \frac{1}{\mu}\frac{h}{F}\frac{100}{R_c(H_o-h_t)}\left(\frac{h_c}{h}\right)^\sigma$$

Reorganizing and integrating will yield

$$\int_{t_t}^{t} dt = \frac{(H_o-h_t)R_c\mu F}{100(h_c)^\sigma}\int_{h_t}^{h} h^{\sigma-1}dt$$

Note t_t = time at $h_t = h$

$$t - t_t = \frac{(H_o-h_t)R_c\mu F}{100(h_c)^\sigma}\left[h^{\sigma-1}\right]_{h_t}^{h}$$

$$t - t_t = \frac{R_c\mu F}{100(h_c)^\sigma}(H_o-h_t)\frac{h_t^\sigma - h^\sigma}{\sigma} \qquad \text{(Eq. A 2)}$$

The total time required to drain a settled sludge to a final head (less than h_t) will be the sum of the times calculated in (Eq. A 1) and (Eq. A 2).

$$\text{Total time} = (t_t) + (t-t_t) = t$$

B. STATISTICAL TEST OF GRAVITY DEWATERING DATA FROM EXPERIMENT IV

To further substantiate the ability of the mathematical model with a media factor of 0.75 to determine the dewatering rates of a wastewater sludge, a statistical analysis of the data was made resulting in the establishment of a confidence interval. This interval was chosen as the upper and lower boundary for 95% of the data. A large interval indicates low correlation and a small interval, high correlation.

It was proven that

$$U = \frac{Y - (a + t^{\circ})}{S_e \sqrt{1 + \frac{1}{n}}}$$

Where: Y = time predicted by mathematical model

a = average time predicted by model minus average time of observed actual drainage

t° = any specific time

n = number of data points

S_e^2 = variance = $\dfrac{\sum_{1}^{n} (y_i - a - t_i)^2}{n - 1}$

y_i = times predicted by model. $i = 1, 2, \ldots n$

t_i = observed actual times. $i = 1, 2, \ldots n$

This equation has a t distribution with (n-1) degrees of freedom. Therefore, the confidence interval of Y is

$$\left(a + t^{*} - \lambda_{\alpha/2}\, S_e \sqrt{1 + \frac{1}{n}}\ ,\ a + t^{*} + \lambda_{\alpha/2}\, S_e \sqrt{1 + \frac{1}{n}} \right)$$

$\lambda_{\alpha/2}$ = a number from a T table that will be exceeded by U, 2.5% of the time, i.e.

$\Pr\{U > \lambda_{\alpha/2}\} = 0.025$

Note: When $n \geq 30$, U will approach a normal distribution, enabling the use of normal distribution tables to determine $\lambda_{\alpha/2}$.

Before confidence intervals for the data from columns 4, 5, 6 and 7, 8, 9 in Experiment IV could be determined, the head loss due to the initial surge at the beginning of the experiment was added to the original data. This could be done since the surge was due to other parameters independent of the mathematical model and was necessitated by the assumptions of the statistical test that all errors be distributed as a random normal variate, $n(0,\sigma^2)$. This same assumption is made for most linear regression analyses.

To meet these statistical assumptions, one centimeter was added to the drainage data from columns 4, 5, 6 and two centimeters to the data from columns 7, 8, 9. The results of the statistical test show that there is very good correlation between the mathematical model and the actual drainage data.

Columns 1, 2, 3	$46.787 < h < 47.275$
Columns 4, 5, 6	$46.720 < h < 47.720$
Columns 7, 8, 9	$46.084 < h < 47.863$

The interpretation of the 95% confidence interval values listed above is that 95% of the time the interval will contain the predicted value (Figure 24). It is apparent that the data collected from columns 1, 2, 3 fell within the 95% confidence interval, thus providing a very high degree of assurance that the mathematical model applies to the data analyzed.

C. MATHEMATICAL PROOF THAT K/L IS A FUNCTION OF WATER CONTENT

The pressure drop of a fluid flowing through a porous media is defined by (Eq. 9) in Chapter IV

$$\frac{\Delta P}{\rho g} = \frac{2(1-\varepsilon)^2\, V_s \mu L S_p^{\,2}}{\varepsilon^3 g V_p^{\,2} \rho} \qquad \text{(Eq. 9)}$$

ΔP = pressure drop ($M\ L^{-1}\ T^{-2}$)

ε = porosity of bed ($M^0\ L^0\ T^0$)

V_s = superficial velocity ($L\ T^{-1}$)

μ = dynamic viscosity of filtrate ($M\ L^{-1}\ T^{-1}$)

L = thickness of media (L)

S_p = surface area of single particle (L^2)

g = acceleration due to gravity ($L\ T^{-2}$)

V_p = volume of single particle (L^3)

ρ = mass density ($M\ L^{-3}$)

Substitution of V_s as $\frac{1}{A}\frac{dV}{dt}$ into (Eq. 9) gives

$$\frac{dV}{dt} = \frac{A\Delta P}{\mu L}\ \frac{V_p^{\,2}}{S_p^{\,2}}\ \frac{\varepsilon^3}{2(1-\varepsilon)^2} \qquad \text{(Eq. A 3)}$$

Making

$$K = \frac{1}{R} = \left(\frac{V_p}{S_p}\right)^2 \frac{\varepsilon^3}{2(1-\varepsilon)^2} \qquad \text{(Eq. A 4)}$$

Where:

R = resistance (L^{-2})

$$\frac{dV}{dt} = \frac{A\Delta P}{\mu L R} = \frac{A\Delta P}{\mu}\left(\frac{K}{L}\right)$$

$$L = \frac{V_{tot}}{A}$$

V_{tot} = total volume of sludge (L^3)

A = cross sectional area (L^2)

Dividing both sides of (Eq. A 4 by $\frac{V_{tot}}{A}$ gives

$$\frac{K}{\frac{V_{tot}}{A}} = \frac{K}{L} = \frac{(V_p/S_p)^2 \epsilon^3}{(1-\epsilon)^2 V_{tot}}$$

$$\epsilon = \frac{V_v}{V_{tot}} = \frac{V_{tot}-V}{V_{tot}}$$

Where:

V = volume of solids (L^3)

V_v = volume of voids (L^3)

$$1-\epsilon = 1 - \left(\frac{V_{tot}-V}{V_{tot}}\right) = \frac{V}{V_{tot}}$$

$$V_{tot} = \frac{V}{1-\epsilon}$$

Since

$V = NV_p$

N = number of particles ($M^0 L^0 T^0$)

V_p = volume of one particle (L^3)

$$V_{tot} = \frac{NV_p}{1-}$$

Therefore

$$\frac{K}{L} = \frac{V_p^{\ 2} \epsilon^3 (1-\epsilon)}{S_p^{\ 2}(1-\epsilon)^2 NV_p}$$

If $B = \frac{V_p}{NS_p^2}$

$$\frac{K}{L} = B\,\frac{\epsilon^3}{1-\epsilon} \qquad \text{(Eq. A 5)}$$

w = Water content = $\frac{100W_w}{W_w + W_{ts}}$

W_w = weight of water

W_{ts} = weight of total solids

Assume that sludge is saturated. Then:

$V_v = \frac{W_w}{\gamma}$ = volume of voids

γ = specific weight of water

$V_{tot} = \frac{W_w + W_{ts}}{\gamma_t}$

γ_t = specific weight of sludge

Since

$$\epsilon = \frac{V_v}{V_{tot}}$$

$$\epsilon = \frac{W_w}{W_w + W_{ts}}\left(\frac{\gamma_t}{\gamma}\right) = \frac{w\gamma_t}{100\gamma}$$

Therefore, substitution in (Eq. A 5)

$$\frac{K}{L} = \frac{B\left(\frac{w\gamma_t}{100\ \gamma}\right)^3}{1 - \frac{w\gamma_t}{100\gamma}}$$

Since

$$\frac{\gamma_t}{\gamma} \simeq \text{unity}$$

$$\frac{K}{L} = \frac{B(w/100)^3}{1 - w/100}$$

APPENDIX II

EXPERIMENTAL DATA

TABLE A 1: SOLIDS CONTENT DATA FROM EXPERIMENTS I-IV

		a	b		
Experiment I	Wet wgt. & pan (gms)	34.2582	31.1863		
	Dry wgt. & pan (gms)	3.4836	3.2990		
	Pan (gms)	1.3958	1.3945		
	Solids content (%)	6.36	6.12		
		a	b		
Experiment II	Wet wgt. & pan (gms)	5.4328	7.8227		
	Dry wgt. & pan (gms)	1.5401	1.6596		
	Pan (gms)	1.3339	1.3337		
	Solids content (%)	5.06	5.01		
		a	b		
Experiment III	Wet wgt. & pan (gms)	8.6211	7.1150		
	Dry wgt. & pan (gms)	1.7312	1.6489		
	Pan (gms)	1.3694	1.3735		
	Solids content (%)	4.98	4.79		
		a	b	c	d
Experiment IV	Wet wgt. & pan (gms)	30.2250	35.2187	33.7359	34.4279
	Dry wgt. & pan (gms)	2.3924	2.5738	2.5606	2.5977
	Pan (gms)	1.3619	1.3463	1.3583	1.3499
	Solids content (%)	3.60	3.66	3.73	3.78
		a	b	c	
Experiment V	Wet wgt. & pan (gms)	33.1110	35.2500	29.5484	
	Dry wgt. & pan (gms)	2.2648	2.3243	2.1873	
	Pan (gms)	1.3468	1.3402	1.3529	
	Solids content (%)	2.89	2.49	3.96	
Experiment VI	Original data missing				
	Solids content (%)	3.50			

TABLE A 2: SAND DATA FROM SIEVE ANALYSIS

Sieve No.	Diameter (mm)	Franklin Sand Wgt. on Sieve (gm)	Franklin Sand % Passing	Ottawa Sand Wgt. on Sieve (gm)	Ottawa Sand % Passing	Hermitage Sand Wgt. on Sieve (gm)	Hermitage Sand % Passing
14	1.410					0	100.0
16	1.190			0	100.0	3.5	98.3
18	1.000						
20	0.841			4.6	96.4	173.5	13.4
25	0.707						
30	0.595			120.8	1.6	24.5	1.5
35	0.500						
40	0.420			2.2		2.9	
45	0.352	0	100.0				
50	0.297	7.40	96.5	0.2			
60	0.250	70.08	63.0				
70	0.210						
80	0.177	88.80	20.5				
100	0.149	32.40	5.0				
140	0.105	8.40	1.0				
200	0.074	1.90					

	Franklin Sand	Ottawa Sand	Hermitage Sand
D_{10} = E = Effective Size	0.16 mm	0.60 mm	0.78 mm
$\frac{D_{60}}{D_{10}}$ = U = Uniformity Coefficient	$\frac{.20}{.16} = 1.25$	$\frac{.74}{.60} = 1.23$	$\frac{1.1}{.78} = 1.41$

TABLE A 3: DRAINAGE DATA FROM EXPERIMENT I

Time Days	Columns 1	2	3	4	5	6	7	8	9
1	62.08	62.14	62.24	79.85	79.78	79.72	98.63	94.82	98.57
2	61.59	61.66	61.79	79.34	79.23	79.21	97.56	88.61	97.83
3	61.27	61.34	61.52	78.98	78.89	78.83	95.40	84.37	91.19
4	57.38	59.65	61.26	76.31	76.62	77.64	94.38	83.35	88.54
5	54.32	54.03	59.41	71.45	68.36	75.26	92.50	82.42	87.25
6	53.68	53.79	54.27	69.08	61.68	72.13	89.98	81.47	86.83
7	53.49	53.68	53.91	66.68	59.53	69.75	87.98	80.50	86.51
8	53.20	53.44	53.68	63.15	57.71	68.92	78.92	79.59	86.16
9	52.96	53.27	53.53	60.27	54.98	67.91	72.09	78.90	85.81
10	52.81	53.12	53.41	57.90	54.28	67.61	66.20	78.27	85.55
11	52.62	52.96	53.20	54.47	54.11	67.30	63.12	77.59	85.20
12	52.50			54.22		67.05	60.97	76.89	84.85
13	52.38			54.09		66.82	60.13	76.40	84.59
14	52.26						59.79	75.92	84.29
15	52.22						59.54	75.33	83.96
16							59.29	74.85	83.66
17							59.06	74.27	83.37
18							58.80	73.67	83.06
19							58.55	73.19	82.77
20							58.29	72.71	82.47

TABLE A 4: DRAINAGE DATA FROM EXPERIMENT II

Time Days	Columns								
	1	2	3	4	5	6	7	8	9
1	47.56	48.98	47.65	66.50	66.38	66.50	87.88	87.78	87.69
2	46.41	48.09	46.57	65.34	65.21	65.31	86.68	86.57	86.52
3	45.59	47.38	45.83	64.53	64.35	64.45	85.77	85.63	85.49
4	41.31	38.81	38.95	63.78	56.72	63.64	85.07	84.81	84.57
5	39.41	38.26	38.12	63.20	38.89	45.22	84.43	84.19	83.93
6	36.97	37.77	37.72	62.69	37.20	41.33	83.93	83.68	83.38
7	36.60	37.47	37.37	59.36	36.84	39.29	83.44	83.20	82.88
8	36.30	37.17	37.05	49.24	36.59	37.86	83.01	82.74	77.64
9	36.00	36.91	36.75	44.90	36.31	36.87	82.64	78.71	59.96
10	35.74	36.67	36.45	43.21	36.05	36.19	68.96	66.82	51.83
11	35.59	36.45	36.22	41.92	35.80	35.74	58.02	55.44	47.35
12	35.29	36.33	36.10	40.85	35.64	35.51	49.15	47.42	43.10
13	34.49	35.51	35.97	40.13	35.50	35.31	45.84	43.26	40.70
14	34.49	35.50	35.12	39.43	34.56	35.09	44.28	40.75	39.65
15				38.71		34.90	43.26	39.33	39.47
16				38.04		34.73	42.39	39.14	39.29

TABLE A 5: DRAINAGE DATA FROM EXPERIMENT III

Time					Columns				
Days	1	2	3	4	5	6	7	8	9
1	57.74	62.02	67.73	66.19	63.27	61.84	67.44	59.88	67.28
2	54.29	52.38	56.91	47.03	53.22	53.51	50.48	45.72	39.89
3	52.50	50.25	54.76	39.95	42.87	40.44	46.20	39.65	39.30
4	51.29	48.58	53.46	39.52	38.14	39.25	43.32	39.18	38.94
5	49.89	47.47	52.38	38.22	37.80	37.70	40.74	37.63	37.27
6	43.82	42.98	50.48	38.22	37.03	37.70	39.34	37.63	37.09
7	38.93	38.27	46.16	38.22	37.03	37.70	37.63	37.63	37.09
8	38.58	37.75	44.38	38.22	37.03	37.70	37.63	37.63	37.09

TABLE A 6: DRAINAGE DATA FROM EXPERIMENT IV

Time Days	Columns 1	2	3	4	5	6	7	8	9
1	52.21	52.07	52.11	51.26	51.14	51.52	50.45	49.71	50.09
2	51.56	51.40	51.44	50.60	50.48	50.93	49.83	49.12	49.47
3	51.02	50.90	50.90	50.07	49.95	50.[illegible]	49.35	48.64	48.94
4	50.54	50.43	50.47	49.68	49.68	50.07	49.00	48.28	48.52
5	50.19	50.07	50.07	49.24	49.12	49.65	48.60	47.87	48.14
6	49.83	49.71	49.65	48.88	48.76	49.29	48.22	47.55	47.72
7	49.47	49.33	49.26	48.55	48.45	49.00	47.90	47.21	47.36
8	49.12	49.00	48.88	48.26	48.16	48.66	47.57	46.88	47.00
9	48.76	48.64	48.57	47.93	47.81	48.40	47.31	46.56	46.71
10	48.52	48.38	48.26	47.66	47.57	48.13	46.97	46.32	46.38
11	48.16	48.02	47.91	47.33	47.21	47.77	46.65	45.98	46.03
12	47.81	47.69	47.57	46.97	46.86	47.45	46.36	45.67	45.70
13	47.57	47.45	47.33	46.74	46.62	47.21	46.14	45.43	45.30
14	47.21	47.09	46.80	46.38	46.26	46.86	45.78	45.07	42.81
15	46.97	46.86	44.33	46.20	46.02	46.62	45.40	44.83	42.45
16	46.80	46.68	42.93	46.00	45.85	45.43	44.48	44.36	42.04
17	46.62	46.44	41.81	45.78	45.67	43.17	42.69	43.17	41.89
18	46.38	46.26	41.62	45.55	45.43	42.86	42.35	41.76	41.68
19	46.20	46.05	41.38	44.12	45.25	42.69	42.21	41.58	41.49
20	46.00	45.84	41.26	42.04	45.07	42.51	42.02	41.41	41.32
21	45.78	45.67	41.08	41.86	44.83	42.33	41.86	41.17	41.08
22	45.61	45.43	40.93	41.68	43.64	42.19	41.68	41.14	40.97
23	45.43	42.33	40.76	41.50	41.86	41.98	41.50	41.02	40.73

TABLE A 6 (Continued): DRAINAGE DATA FROM EXPERIMENT IV

Time	Columns								
Days	1	2	3	4	5	6	7	8	9
24	45.38	41.98	40.55	41.38	41.68	41.80	41.31	40.91	40.61
25	41.86	41.74	40.43	41.16	41.50	41.62	41.14	40.73	40.39
26	41.36	41.56	40.19	40.97	41.31	41.44	40.91	40.61	40.25
27	41.14	41.30	40.00	40.79	41.14	41.26	40.73	40.43	40.01

TABLE A 7: DRAINAGE DATA FROM EXPERIMENT V

Time	Columns								
Days	1	2	3	4	5	6	7	8	9
1	40.96	40.07	37.45	70.04	68.85	67.96	110.19	108.94	105.69
2	39.95	39.12	36.32	68.97	67.72	66.91	108.95	107.69	104.48
3	38.16	37.93	35.67	68.14	66.90	66.06	107.93	106.62	103.49
4	37.53	37.33	35.19	67.31	65.88	65.28	106.97	105.67	102.50
5	37.02	36.82	34.49	66.59	65.05	64.69	106.14	104.71	101.55
6	36.58	35.67	34.24	65.88	64.31	63.98	105.31	103.86	100.72
7	36.26	35.55	34.19	65.17	63.59	63.32	104.48	103.02	99.90
8	35.07	35.44	34.19	64.57	63.12	62.67	103.57	102.45	99.17
9			34.17	63.80	62.70	61.95	102.74	101.83	98.34
10			34.12	63.02	62.41	61.24	101.86	101.39	97.61
11				62.21	62.19	60.70	101.38	100.93	97.09
12				61.51	61.83	60.41	101.02	100.55	96.67
13				61.38	61.48	60.17	100.61	100.07	96.21
14				61.12	61.12	59.93	100.19	99.66	95.84
15				60.88	60.76	59.68	99.84	99.24	95.48
16				60.58	60.52	59.43	99.43	98.85	95.12
17				60.34	60.29	59.22	99.03	98.41	94.68
18				60.13	59.93	58.98	98.65	97.96	94.29
19									
20				59.17	59.45	58.55	97.93	97.22	93.58
21				58.93	49.16	58.32	97.57	96.80	93.34
22				58.57	58.74	57.91	96.90	96.05	92.66
23				58.52	58.50	57.67	96.62	95.67	92.20
24				58.20	58.32	57.54	96.27	95.33	91.84

TABLE A 7 (Continued): DRAINAGE DATA FROM EXPERIMENT V

Time Days	Columns 1	2	3	4	5	6	7	8	9
25				58.05	58.03	57.31	95.91	94.96	91.46
26				57.80	57.79	57.12	95.56	94.60	91.24
27				57.57	57.55	56.96	95.19	94.20	90.85
28				57.13	57.31	56.72	94.85	93.85	90.39
29				56.99	57.07	56.57	94.50	93.41	90.04
30				56.74	56.84	56.37	94.16	92.97	89.74
31				56.62	56.60	56.21	93.83	92.56	89.41
32				56.50	56.37	56.02	93.50	92.11	89.20
33				56.27	56.17	55.86	93.17	91.70	88.82
34				56.03	56.00	55.71	92.85	91.32	88.48
35					55.77	55.53	92.52	90.91	88.25
36					55.49	55.29	92.18	90.40	87.56
37					55.27	55.17	91.83	89.90	87.28
38					55.10	55.03	91.51	89.51	86.96
39					54.91	55.77	91.21	89.09	86.66
40					54.72	54.58	90.91	88.71	86.23
41									
42					54.34	54.34	90.20	87.97	85.75
43					54.10	54.10	88.80	87.54	84.88
--									
46					53.62	53.61	88.65	86.44	84.34
47					53.45	53.50	88.35	86.07	78.51
--									
55					52.11	52.21	85.93	83.27	72.97
--									
57					51.72	51.90	85.27	83.14	

TABLE A 8:

DRAINAGE DATA WITH PIEZOMETER READINGS FROM EXPERIMENT VI

Time		Piezometer Tubes									
Days	Head	1	2	3	4	5	6	7	8	9	10
1	93.17	94.7	94.9	95.0	95.2	95.3	95.4	95.3	95.5	95.4	2.8
2	92.22	93.5	93.5	93.7	93.6	94.2	94.2	94.2	93.3	94.4	2.7
3	91.64	92.6	92.6	93.1	93.2	93.3	93.2	93.3	90.9	92.9	2.6
4	91.08	91.8	92.3	92.3	92.4	92.4	92.6	92.6	90.3	92.6	2.3
5	90.59	91.5	91.8	91.9	92.0	92.0	92.1	92.1	87.9	92.1	2.5
6	90.16	91.0	91.3	91.4	91.5	91.6	91.6	91.6	86.3	91.5	2.3
7	89.73	90.3	90.4	90.4	90.5	90.5	90.6	90.6	84.7	90.3	2.0
8	89.36	89.7	89.7	89.7	89.9	90.0	90.0	89.8	83.7	88.9	2.4
9	89.00	90.7	90.6	90.6	90.9	90.9	90.9	90.2	84.0	89.9	3.6
10											
11	88.25	89.1	89.4	89.4	89.6	89.6	89.6	89.5	82.0	89.1	2.9
12	87.90	89.5	89.7	89.7	89.9	89.9	89.9	89.4	82.2	89.2	3.6
13	87.50	88.8	88.9	88.9	89.1	89.1	89.1	89.2	81.5	88.6	3.3
14											
15	86.90	86.8	87.0	87.0	87.0	87.0	86.9	86.5	79.0	86.3	1.9

APPENDIX III

GRAPHICAL REPRESENTATION OF EXPERIMENTAL DATA

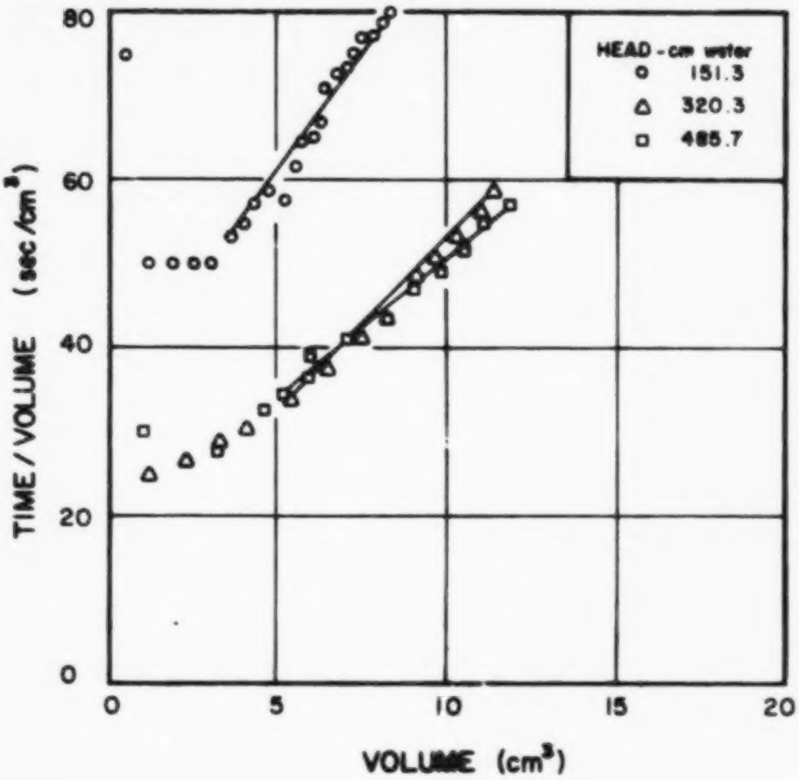


Figure A 1: Specific resistance determinations for Experiment I.

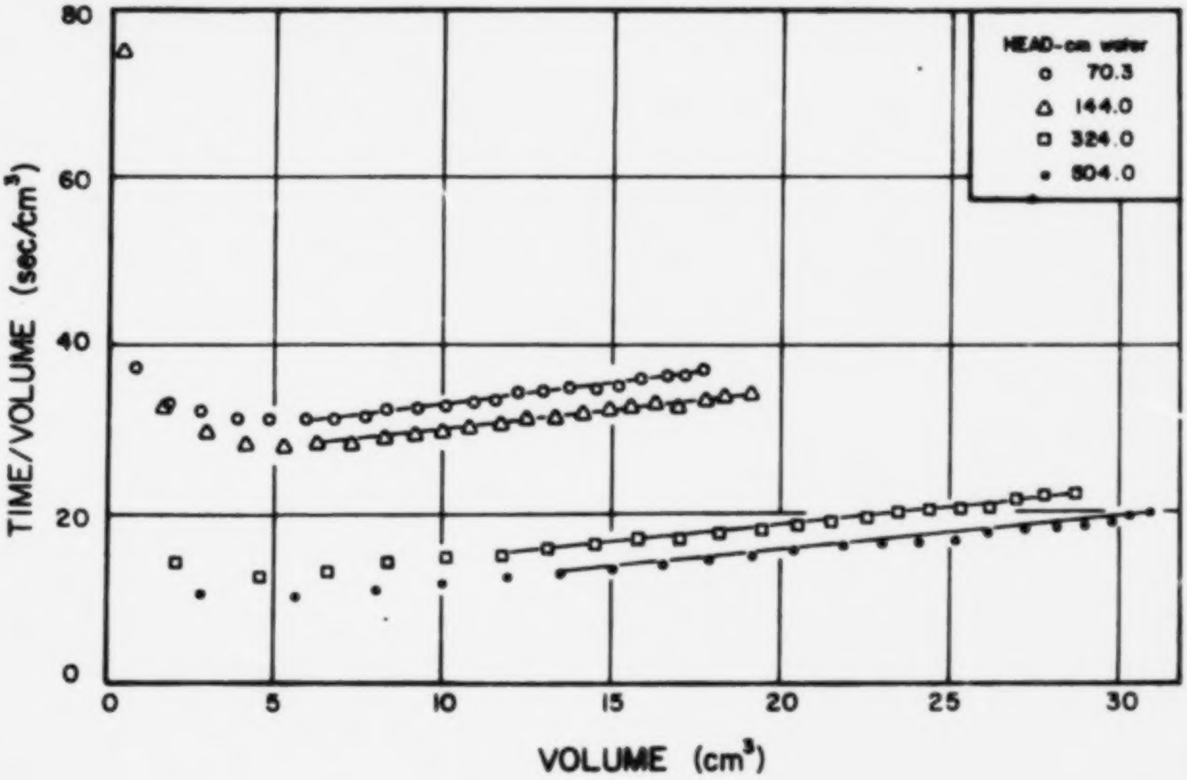


Figure A 2: Specific resistance determinations for Experiment II.

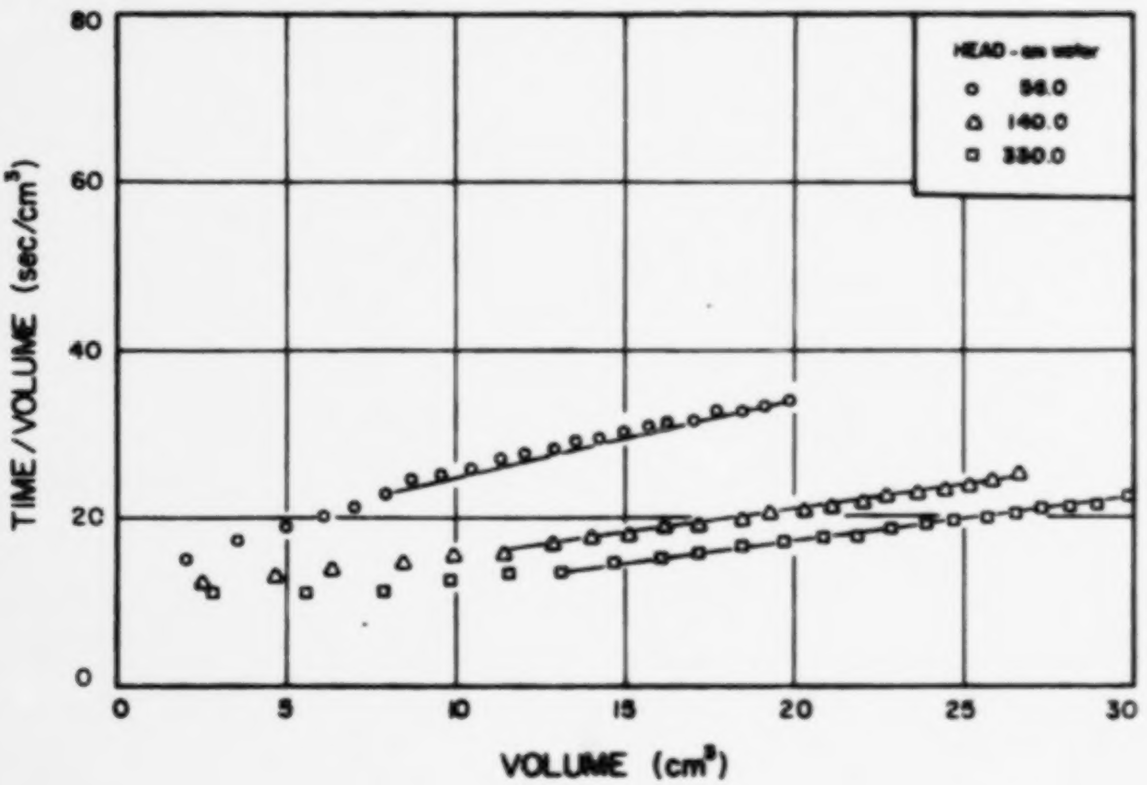


Figure A 3: Specific resistance determinations for Experiment III.

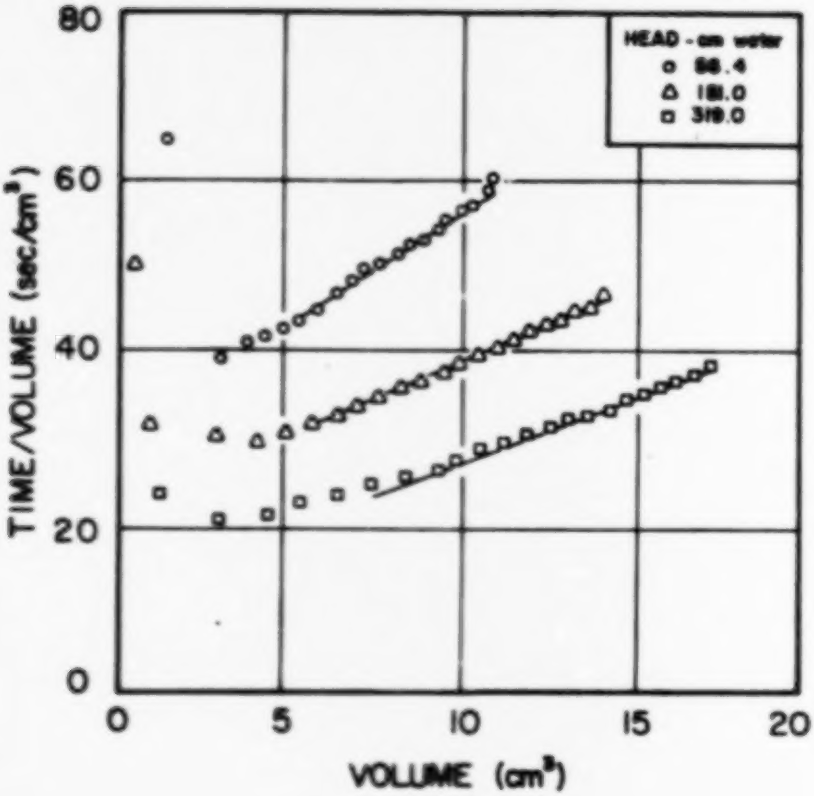


Figure A 4: Specific resistance determinations for Experiment IV.

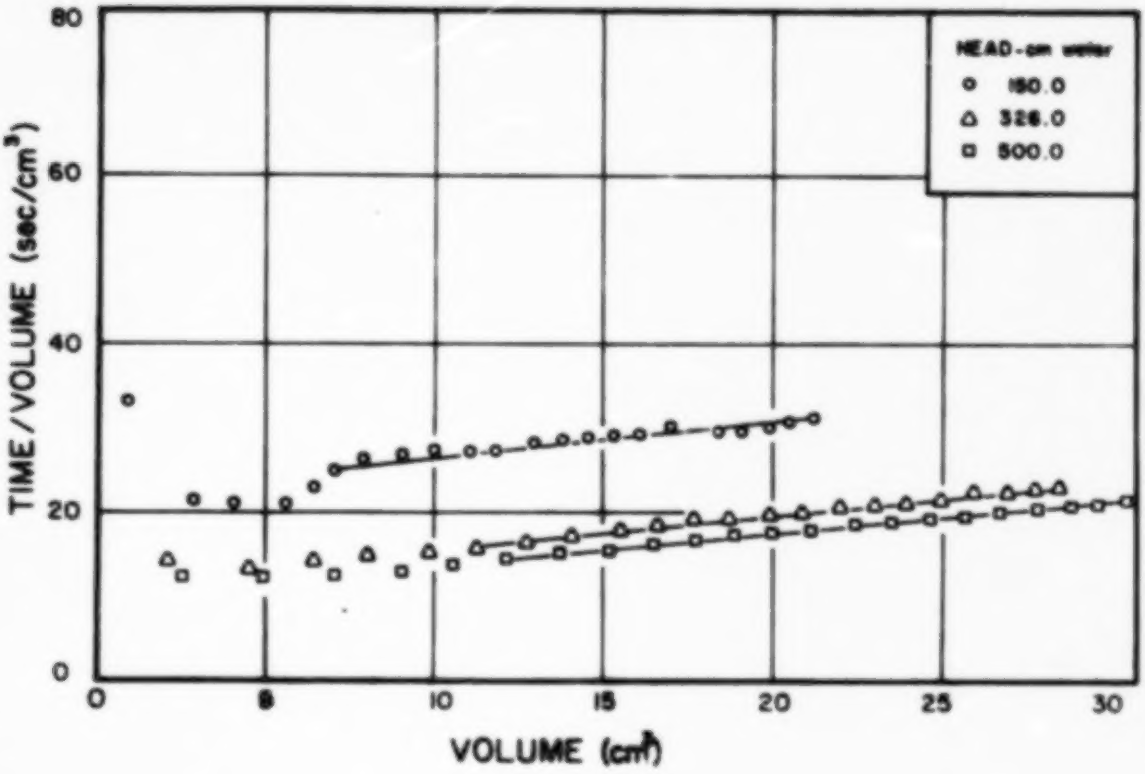


Figure A 5: Specific resistance determinations for Experiment V.

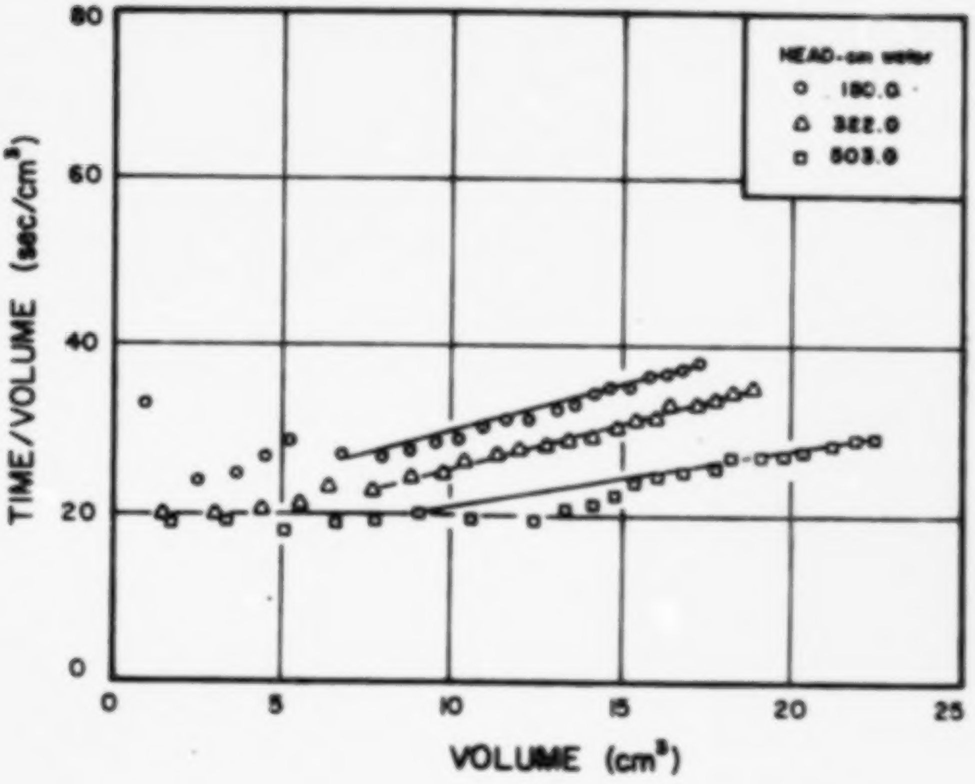


Figure A 6: Specific resistance determinations for Experiment VI.

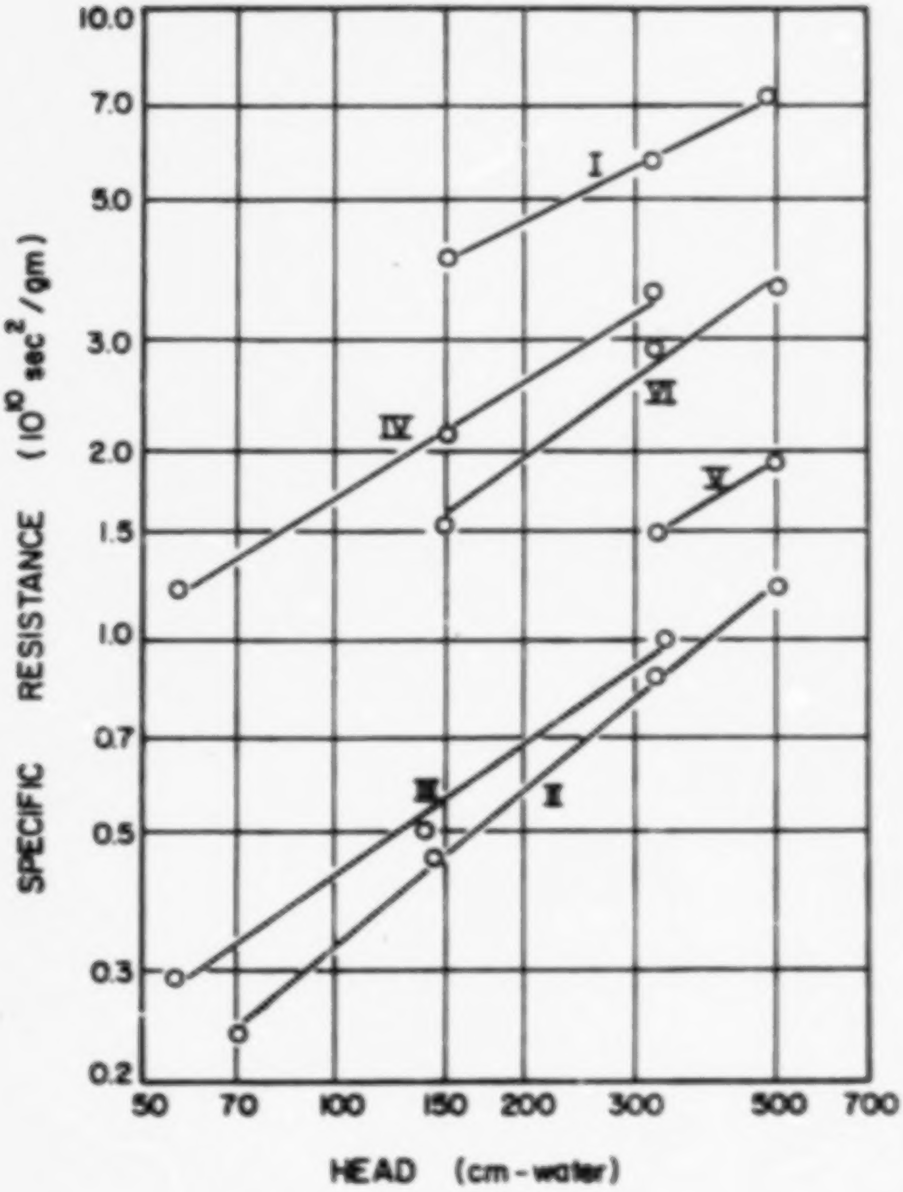


Figure A 7: Coefficient of compressibility determinations for Experiments I-VI.

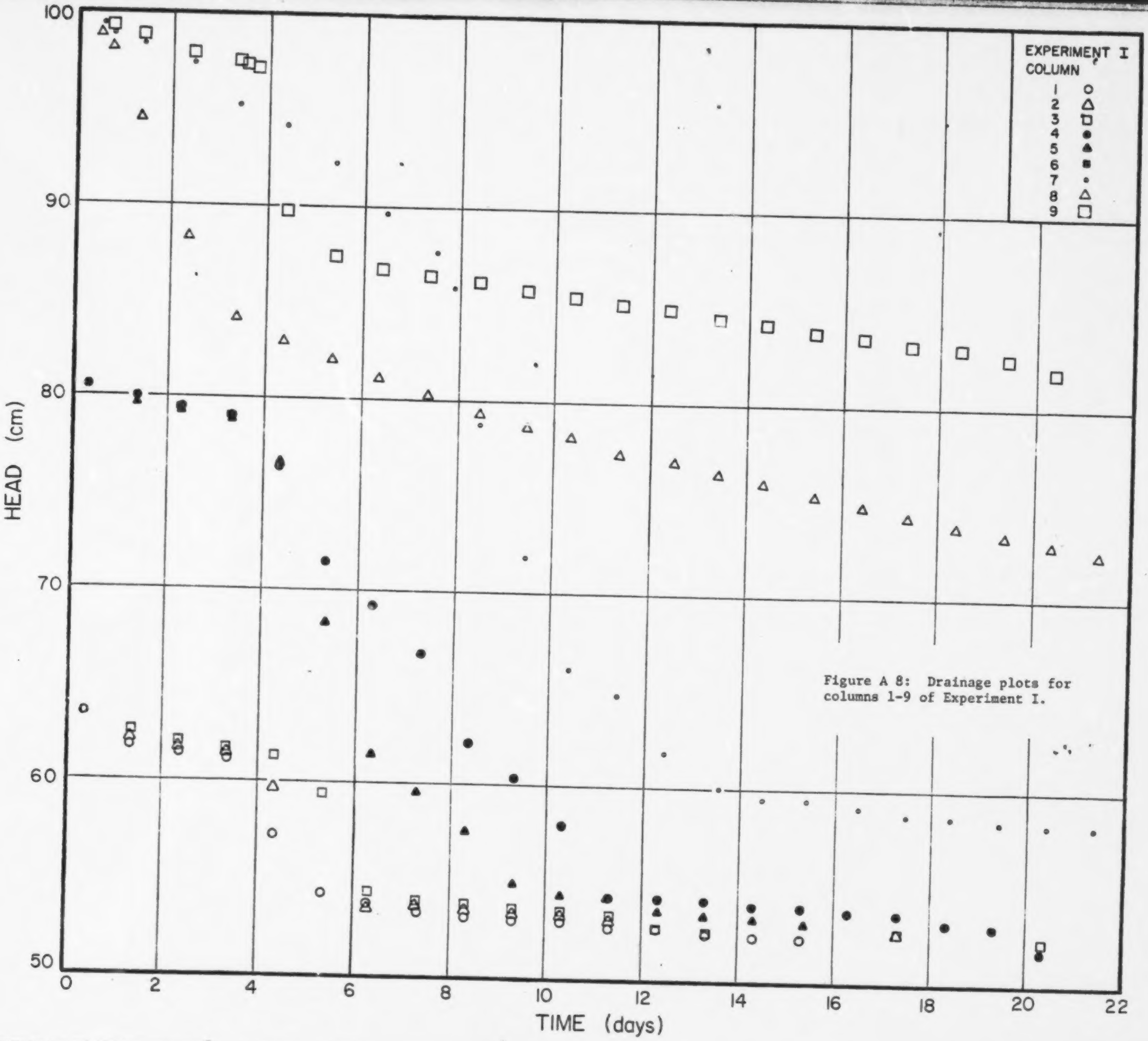


Figure A 8: Drainage plots for columns 1-9 of Experiment I.

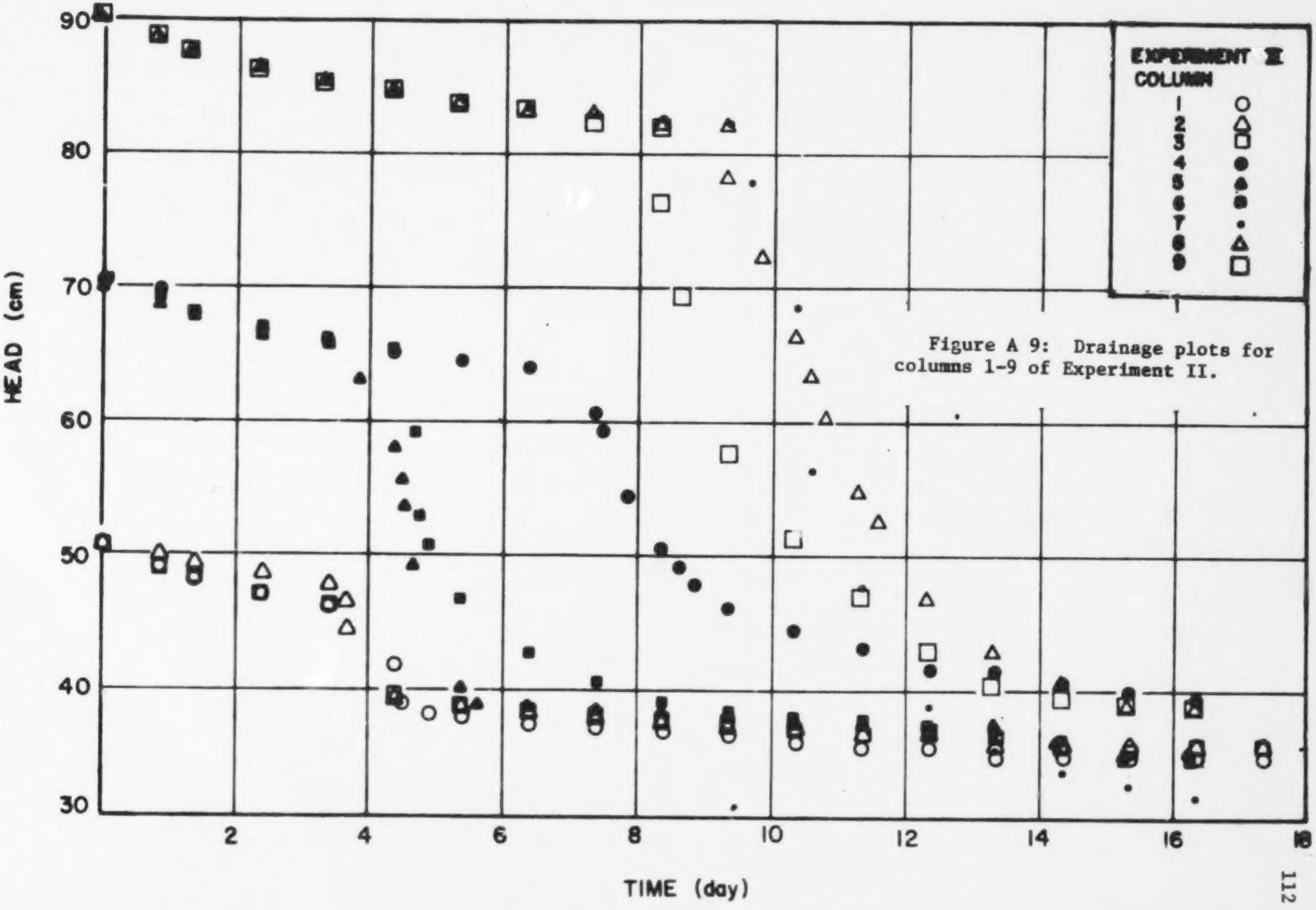


Figure A 9: Drainage plots for columns 1-9 of Experiment II.

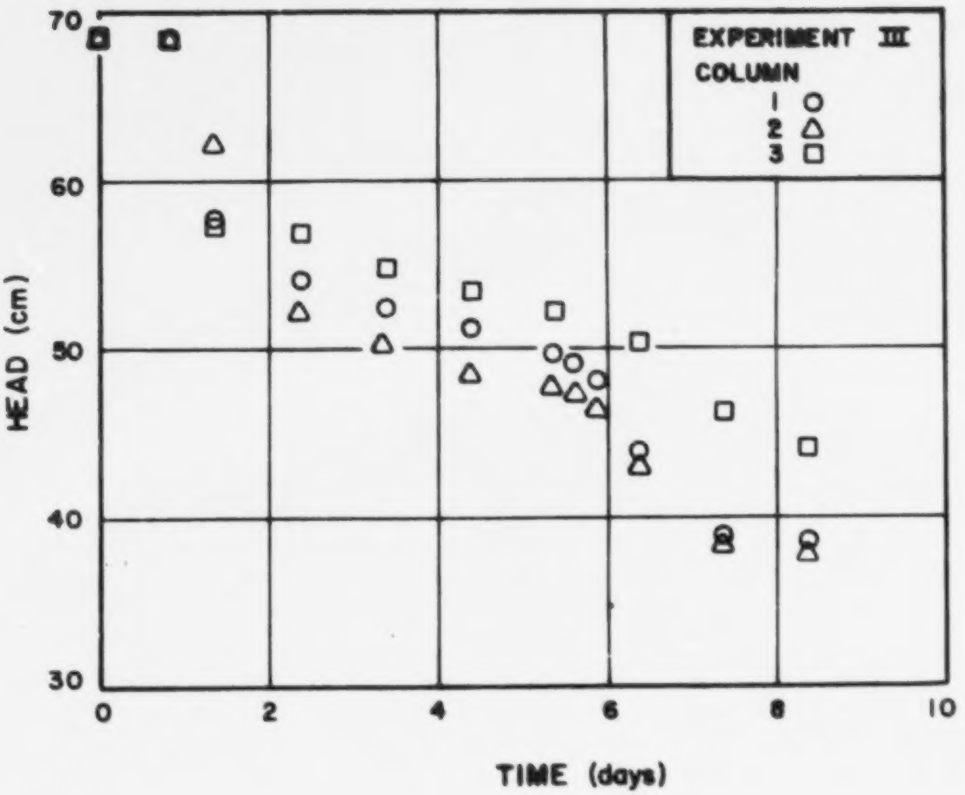


Figure A 10: Drainage plots for columns 1-3 of Experiment III.

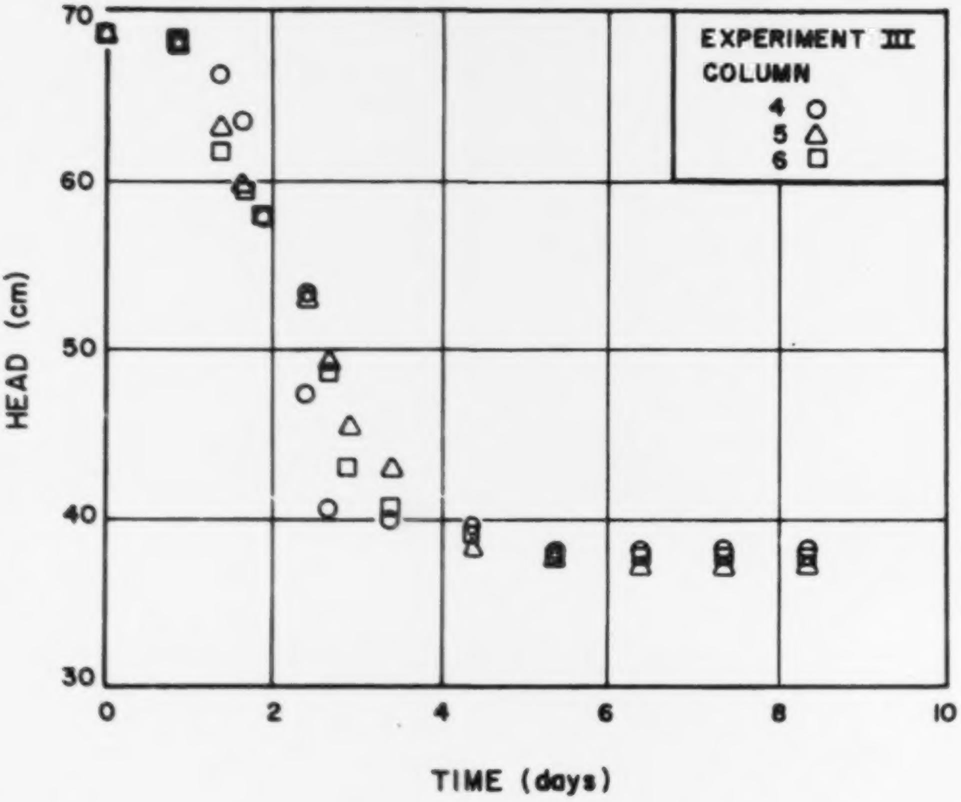


Figure A 11: Drainage plots for columns 4-6 of Experiment III.

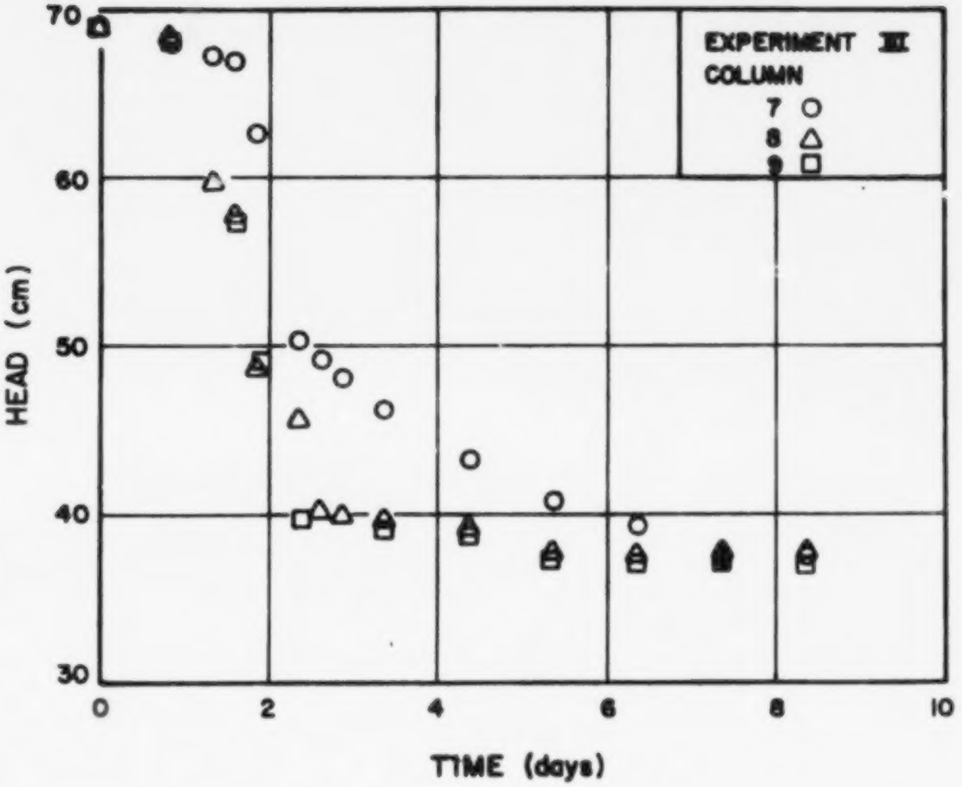


Figure A 12: Drainage plots for columns 7-9 of Experiment III.

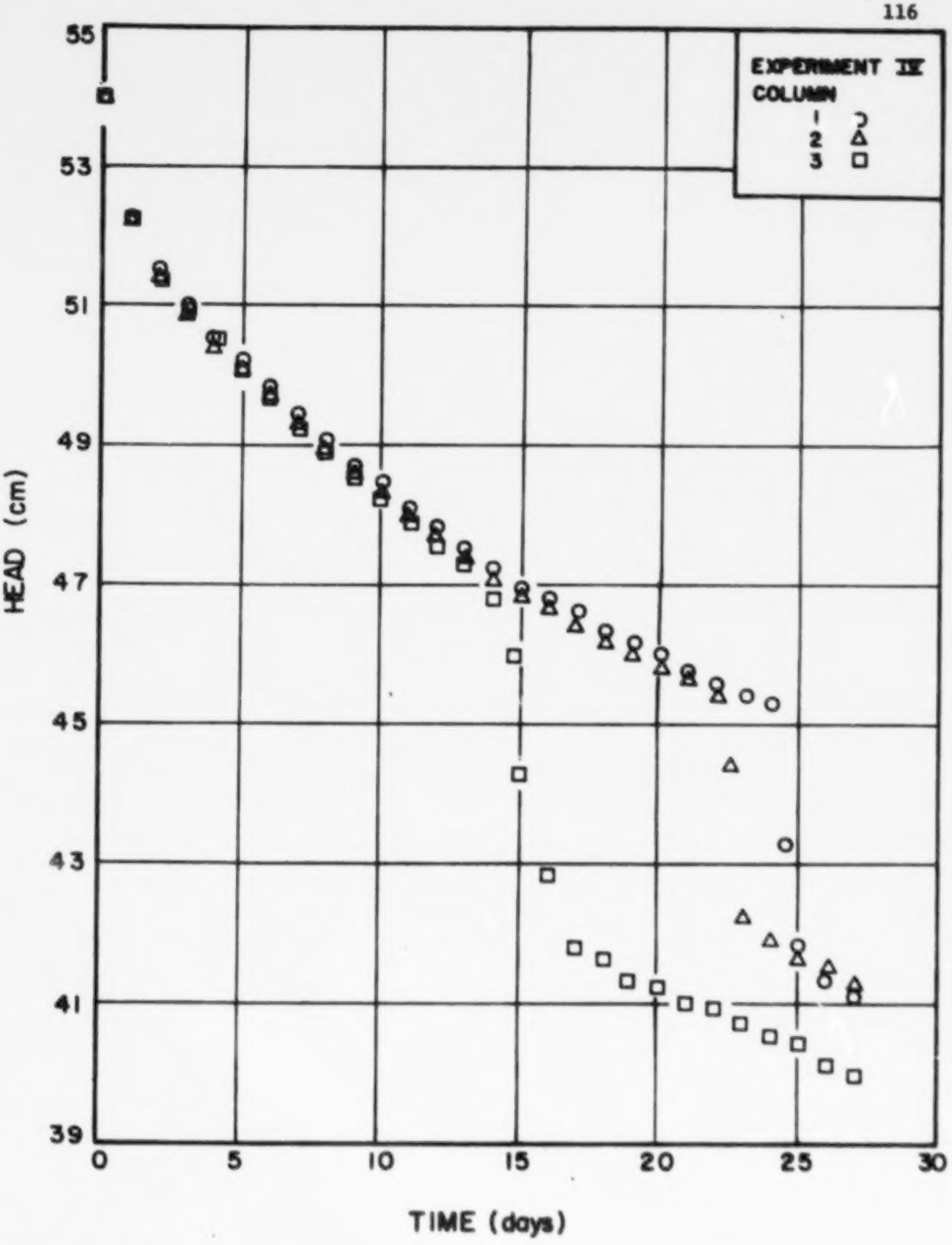


Figure A 13: Drainage plots for columns 1-3 of Experiment IV.

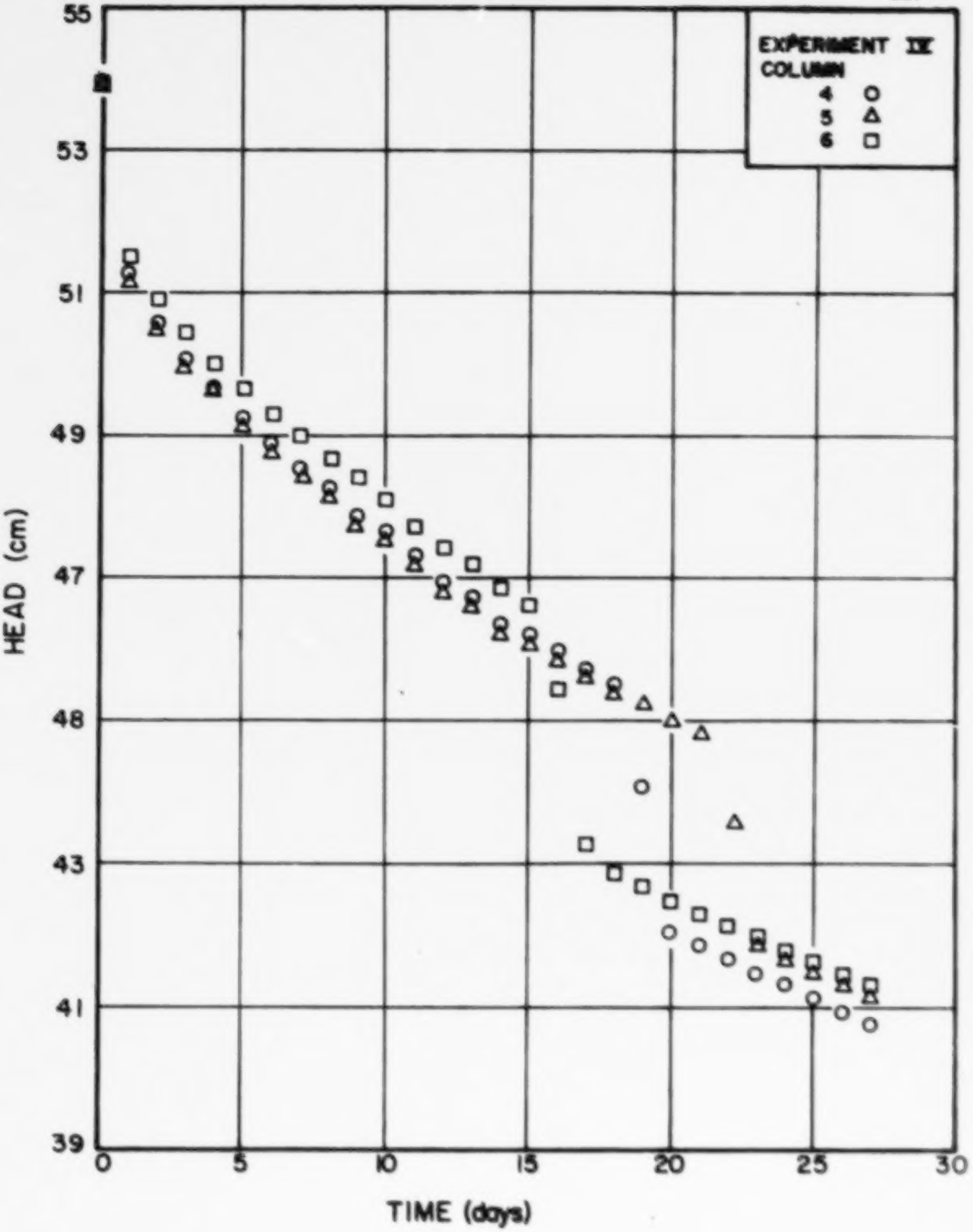


Figure A 14: Drainage plots for columns 4-6 of Experiment IV.

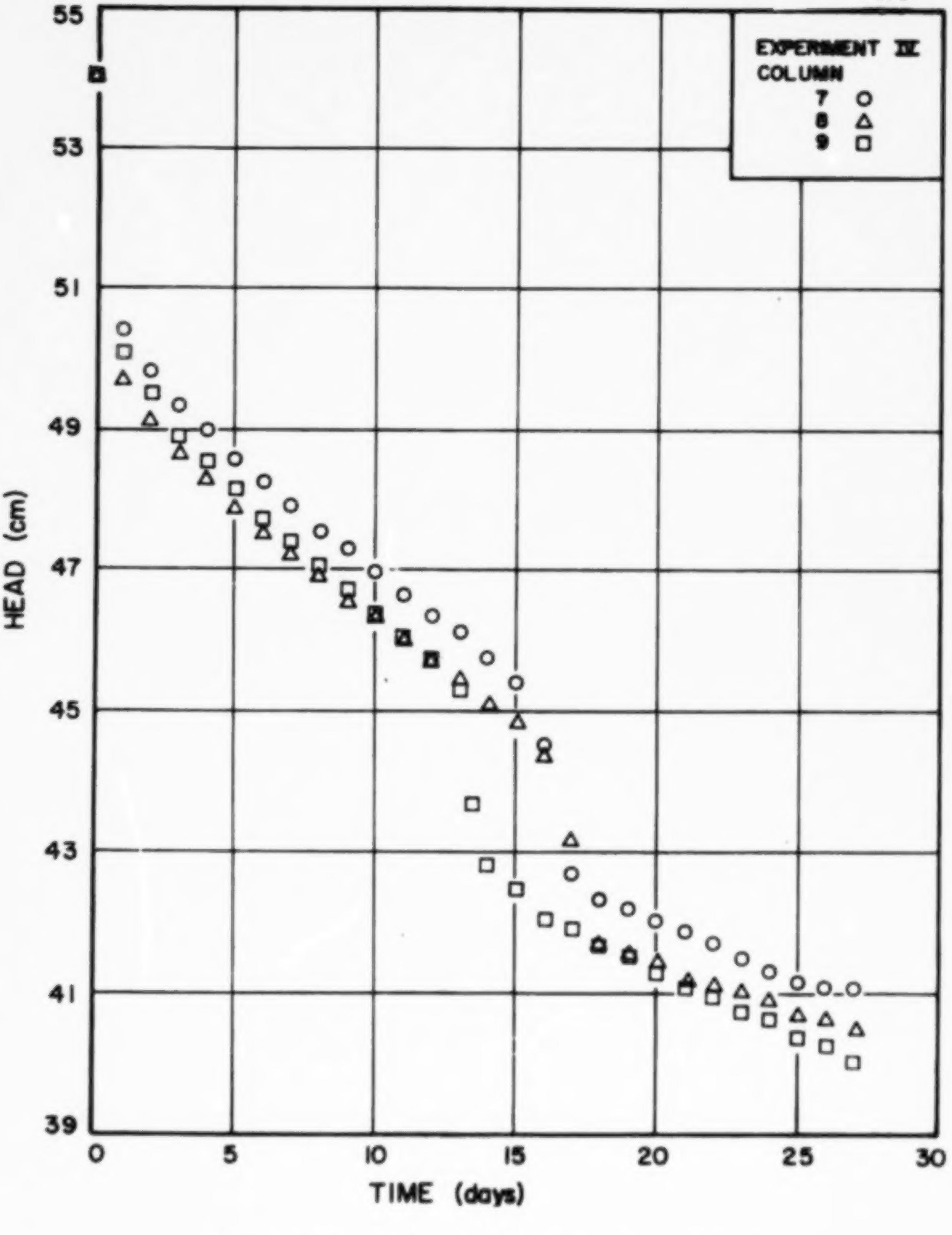


Figure A 15: Drainage plots for columns 7-9 of Experiment IV.

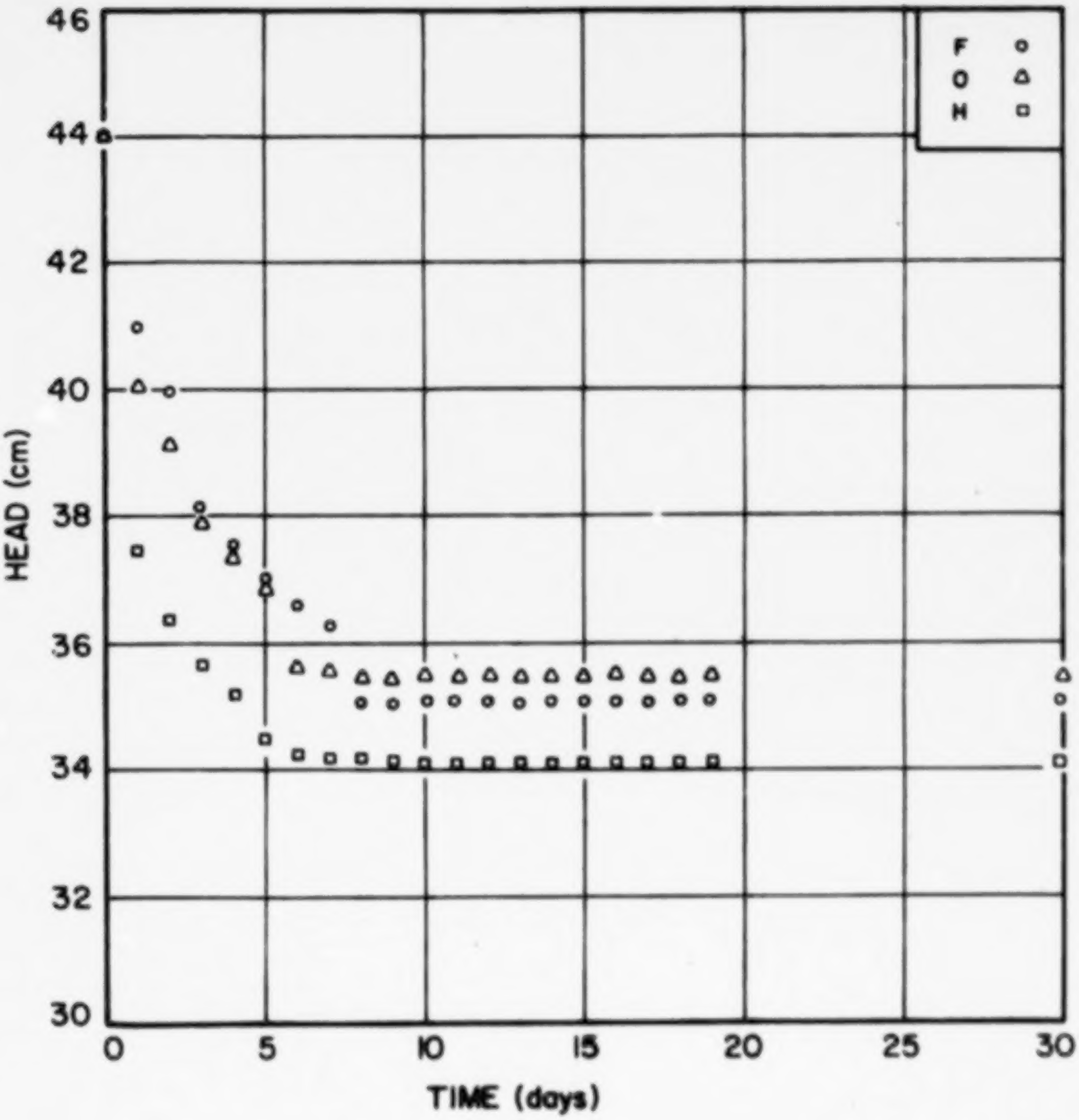


Figure A 16: Drainage plots for columns 1-3 of Experiment V. The symbols F, O, and H refer to the three sand types used.

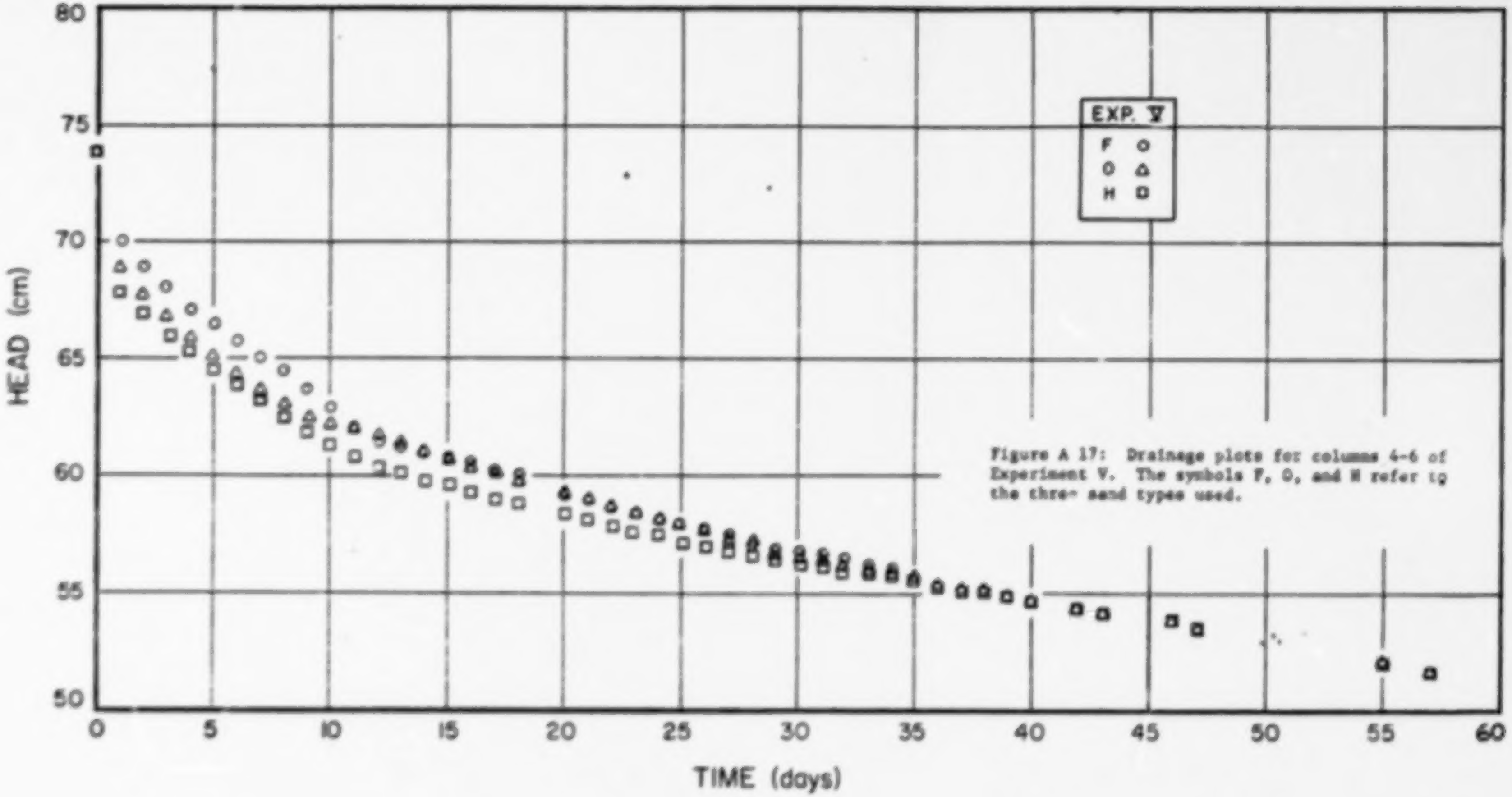


Figure A 17: Drainage plots for columns 4-6 of Experiment V. The symbols F, O, and H refer to the three sand types used.

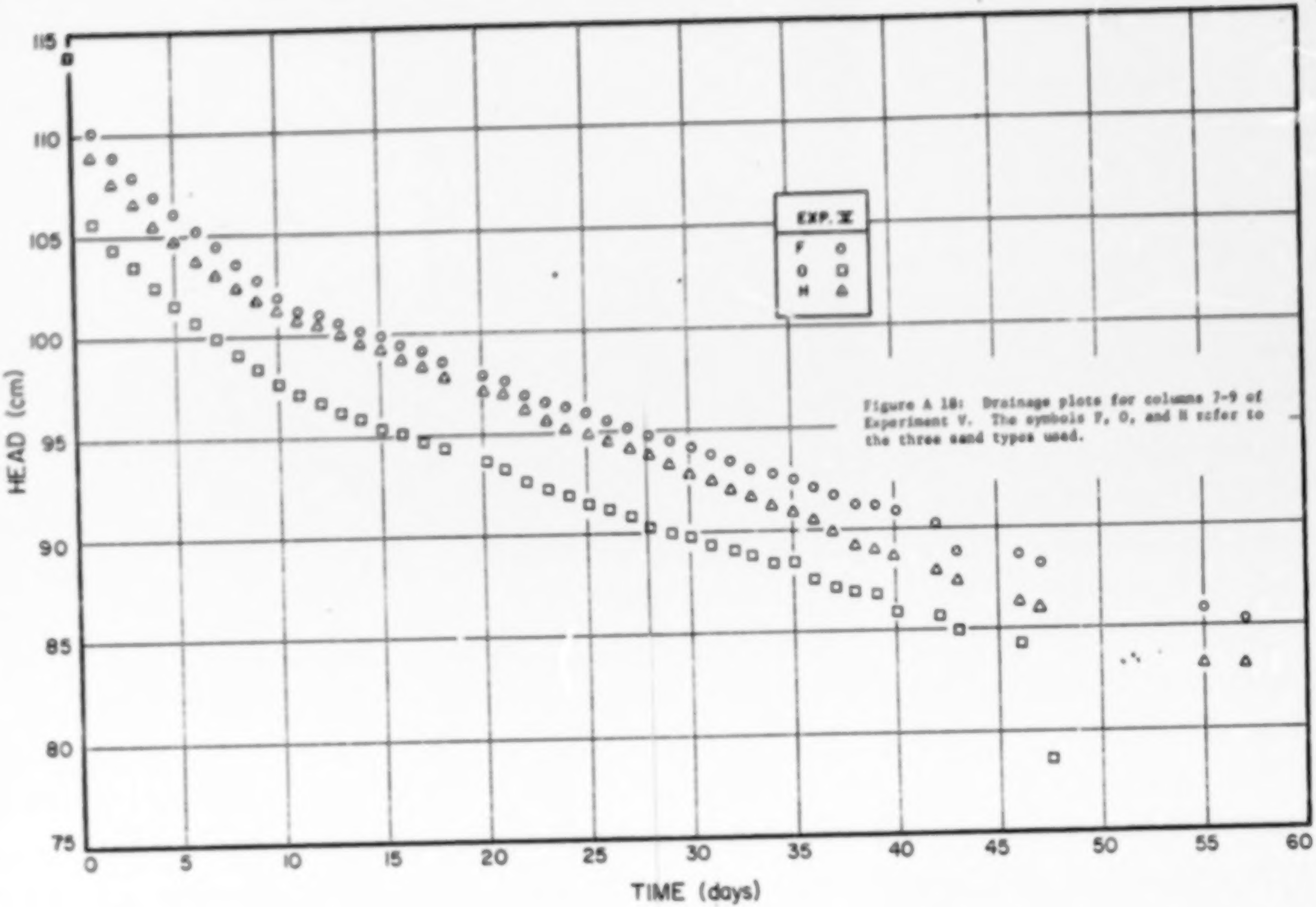


Figure A 18: Drainage plots for columns 7-9 of Experiment V. The symbols F, O, and H refer to the three sand types used.

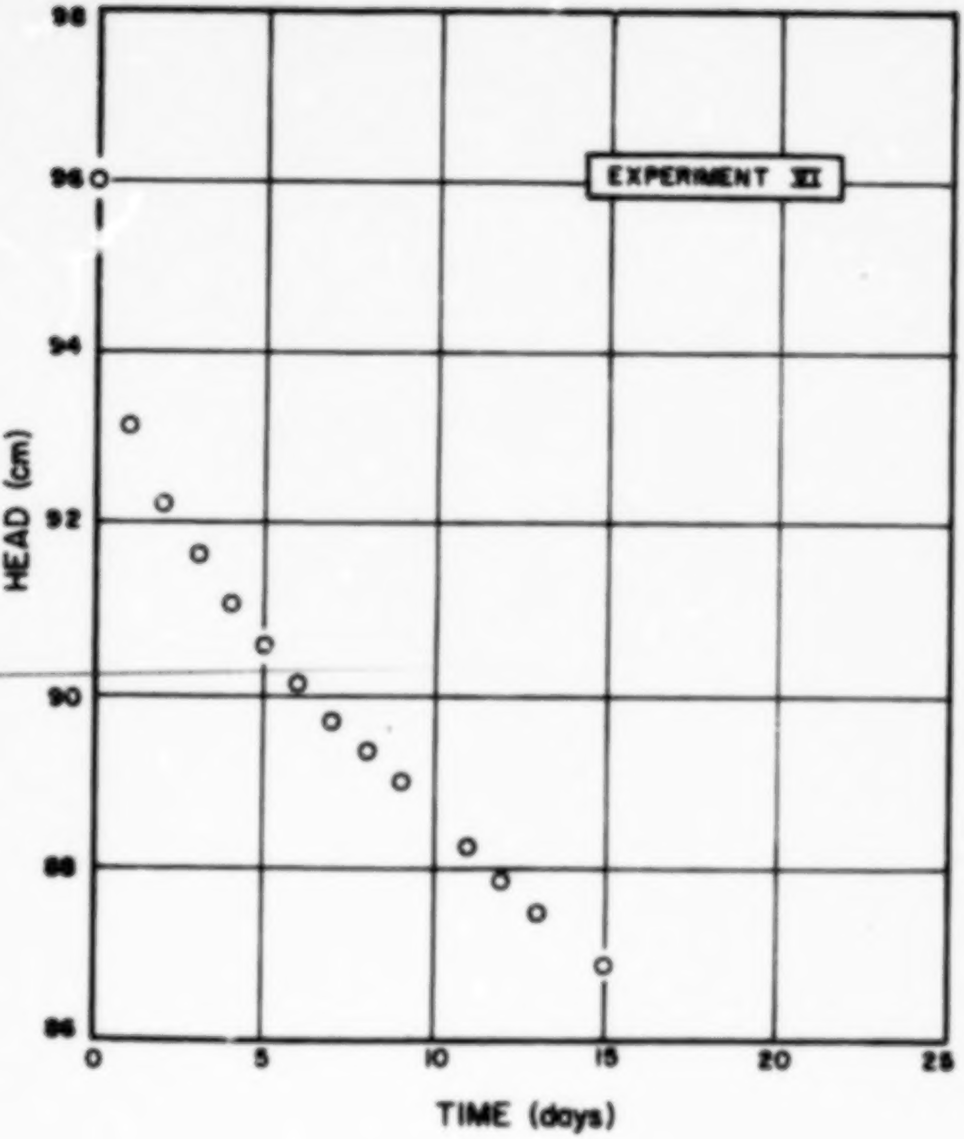


Figure A 19: Drainage plot~~s~~ for ~~column [illegible] of~~ Experiment VI.

```
$      202 51372 5 02 TOM SANDERS
$$$    COMPILE.FORTRAN.
C        SLOPE DETERMINATION FOR SPECIFIC RESISTANCE
         DIMENSION S(22).T(22).Q(22).NNX(110).Y(50).Z(50).AA(50).B(50)
         DO 10 NN=1.110
      10 NNX(NN)=0
         READ(1.2)(S(M).M=1.22)
       2 FORMAT(11F6.2)
     309 READ(1.8)(T(M).M=1.22)
       8 FORMAT(11F5.1)
         IF (T(1))800.999.800
     800 WRITE OUTPUT TAPE 5.600
     600 FORMAT(40X.11HTIME/VOLUME//)
         DO 30 M=1.22
         Q(M)=S(M)/T(M)
         NN=Q(M)
         NNX(NN)=2626262626
         IF (M-1)201.400.201
     201 JA=1
         XM=(T(M)-T(M-1))*10.0
         MM=XM
         MX=1
     301 IF (MM-MX)300.400.300
     300 WRITE(5.302)JA
     302 FORMAT(I1)
         MX=MX+1
         GO TO 301
     400 WRITE(5.3)T(M).(NNX(I).I=1.NN)
       3 FORMAT(F6.2.2X110A1)
      30 NNX(NN)=0
         SL=(Q(22)-Q(6))/(T(22)-T(6))
         WRITE(5.6)SL
       6 FORMAT(30X.6HSLOPE=F8.4///)
         WRITE OUTPUT TAPE 5.601
     601 FORMAT(34X.4HTIME.12X.6HVOLUME.8X.11HTIME/VOLUME//)
         DO 17 M=1.22
         WRITE(5.7)S(M).T(M).Q(M)
       7 FORMAT(30X.5HS(M)=F6.2.5X.5HT(M)=F6.2.5X.5HQ(M)=F6.2)
      17 CONTINUE
         READ(1.50)P.A.E.C
      50 FORMAT(4F12.6)
         R=(2.0*SL*P*A**2)/(E*C)
         WRITE(5.93)P.A.E.C
      93 FORMAT(30X.2HP=F10.5.5X.2HA=F10.5.5X.2HE=F10.5.5X.2HC=F10.5//)
         WRITE(5.90)R
      90 FORMAT(30X.20HSPECIFIC RESISTANCE=F15.2.12H (SEC)**2/GM/)
         Y(5)=0.0
         Z(5)=0.0
         DO 100 M=6.22
         Y(M)=Y(M-1)+T(M)
         Z(M)=Z(M-1)+Q(M)
     100 CONTINUE
```

```
      SAVE=Y(22)/16.0
      ZAVE=Z(22)/16.0
      AA(5)=0.0
      B(5)=0.0
      DO 200 M=6.22
      AA(M)=(T(M)-SAVE)*(Q(M)-ZAVE)+AA(M-1)
      B(M)=(T(M)-SAVE)**2+B(M-1)
  200 CONTINUE
      SL2=AA(22)/B(22)
      WRITE(5.102)SL2
  102 FORMAT(30X.4HSL2=F10.5)
      W1=(2.0*SL2*P*A**2)/(E*C)
      WRITE(5.90)W1
      GO TO 300
  999 CALL EXIT
      END
 30.0 60.0  90.0  120.  150.  180.  210.  240.  270.  300.  330.
 360. 390.  420.  450.  480.  510.  540.  570.  600.  630.  660.
```

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